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AFWAL-TR-81-4097

REFERENCE DOCUMENT FOR THE ANALYSIS OF CREEP AND STRESS-RUPTURE DATA IN MIL-HDBK-5

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September 1981

Interim Report for Period June 1980 - June 1981

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFWAL-TR-81-4097	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) REFERENCE DOCUMENT FOR THE ANALYSIS OF CREEP AND STRESS RUPTURE DATA IN MIL-HDBK-5	5. TYPE OF REPORT & PERIOD COVERED Interim Report June 1980 - June 1981
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Richard C. Rice, Editor	8. CONTRACT OR GRANT NUMBER(s) F33615-80-C-5037
9. PERFORMING ORGANIZATION NAME AND ADDRESS Battelle's Columbus Laboratories 505 King Avenue Columbus, Ohio 43201	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS FY1457-81-04013
11. CONTROLLING OFFICE NAME AND ADDRESS DCASMA, Dayton - Defense Logistics Agency, Attention: DCRO-GDCA, c/o Defense Electronics Supply Center, Dayton, Ohio 45444	12. REPORT DATE September 1981
	13. NUMBER OF PAGES 62
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Materials Laboratory (AFWAL/MLSA) Air Force Wright Aeronautical Laboratories (AFSC) Wright-Patterson Air Force Base, Ohio 45433	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

creep
stress rupture
design of experiments
regression analysis
analysis of variance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report provides background information on creep and stress rupture analysis which expands on the methods described in MIL-HDBK-5 guidelines. The document includes discussions on (1) design of experiments for the purpose of developing regression models, (2) important correlative information for use in evaluating elevated temperature property data, and (3) a comprehensive method of rupture data analysis.

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PREFACE

This interim report was compiled by Battelle's Columbus Laboratories, 505 King Avenue, Columbus, Ohio 43201, under contract F33615-80-C-5037 with the Air Force Wright Aeronautical Laboratory, Wright-Patterson Air Force Base, Ohio, as a part of the ongoing MIL-HDBK-5 coordination effort. Mr. C. L. Harmsworth (MLSA) was the project monitor. This report was submitted by the author, Mr. Richard Rice, in June 1981.

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STATISTICAL GLOSSARY

(1) Analysis of Variance. The analysis of the total variability of a set of data (measured by their total sum of squares) into components which can be attributed to different sources of variation. For example, scatter measurements can be proportioned into (a) heat-to-heat and (b) repeatability, or within heat scatter.

(2) Distribution Analysis determines (a) the underlying distribution, and (b) the parameters of that distribution.

(3) Log-Log Plots are used to graph observations drawn from a distribution whose underlying density function is log normal, Weibull, or extreme value. Data from these distributions will graph a straight line. An example would be stress rupture life in hours as a function of stress.

(4) Log-Normal Distribution is a density function that is not symmetrical, but is positively skewed. If the logarithms of the values of a random variable have a normal distribution, the random variable itself is said to have the log-normal distribution. Creep and stress rupture values in hours are just a few of the phenomena that may be log-normally distributed.

(5) Normal Distribution is a probability law called the normal density function and can be defined mathematically with parameters $\bar{x}$ (mean) and σ (scatter measurement). The graph of this function is a symmetric bell-shaped curve. The normal distribution forms the cornerstone of a very large portion of statistical theory.

(6) Outlier(s) are observations at either extreme (small or large) of a sample which are so far removed from the main body of the data that the appropriateness of including them in the sample is questionable. There are statistical methods to determine the probability that the extreme value observed is an outlier.

(7) Regression Model is the mathematical model chosen to represent the universe that applies to the distribution from which the observations were drawn. Regression models are developed by three methods:

(1) When the model is known in advance, regression analysis derives the coefficients for the model.

(2) Step wise by adding terms representing the independent variables and then testing to see if they have made a significant improvement in the model.

(3) Backward elimination, where every term thought to be significant is put into a model and then terms are eliminated by removing the least significant terms after each regression analysis.

GLOSSARY (concluded)

(8) Residual is the difference between the observed value and the corresponding fitted or predicted value. Residuals are highly useful for studying whether a given regression model is appropriate for the data at hand. They can be used to locate areas of inconsistency in model building. The sum of the residuals is zero.

(9) Scatter Measurements are estimates made on the dispersion of observations about a mean. This measurement is called the standard deviation and symbolized with the Greek letter σ. It is often referred to as the repeatability error.

(10) Slope Measurements are observations made about isothermal regression lines relating life as a function of stress. These observations are made to estimate the slope of these lines.

(11) Standard Error of Estimate (SEE) is a measure of the scatter of observations about a regression line. It is the square root of the residual variance. A mathematical derivation is available in any text on regression analysis.

(12) Tolerance Intervals, also referred to as tolerance limits, are the intervals or limits within which 100 $(1-\alpha)$ % of future observations are expected to fall. The width of these intervals is a function of the size of the standard deviation and the degree of uncertainty resulting from making estimates of the mean and the standard deviation from a finite sample.

LIST OF SYMBOLS

b_i	=	regression coefficients
BSSD	=	between-heat sum of squared deviations
c	=	intercept term in the regression = b_0
c_h	=	intercept term in the regression for a given heat of an alloy
$\overline{C}$	=	arithmetic average of the c_h's
f	=	degrees of freedom
H	=	number of heats of an alloy
h	=	heat index
k	=	the number of parameters in the regression, or the magnitude of the noncentral statistic
m	=	thickness
MSB	=	between-heat mean square
MSW	=	within-heat mean square
N	=	total number of data values
n_i	=	sample size of ith heat
$\overline{n}$	=	weighted average number of observations per heat
R^2	=	coefficient of multiple determination
S_i	=	sample sum of ith heat
s_i	=	standard deviation of ith heat
SEE	=	standard error of estimate
SS_i	=	sum of squares of ith heat
SSD	=	sum of squared deviations
SSE	=	sum of squares error
T	=	temperature, degrees Fahrenheit
T_A	=	temperature of convergence of the iso-stress lines

LIST OF SYMBOLS (concluded)

t	=	time, hours
TSSD	=	total sum of squared deviations
V_B	=	between-heat variance
V_h	=	variance of heat intercept
V_T	=	total variance
V_w	=	within-heat variance
W_i	=	weight for the ith heat used to calculate regression intercepts when separating heats
WSSD	=	within-heat sum of squared deviations
x	=	independent variable
$\overline{x}$	=	average of the independent variable
x_{ij}	=	jth observation in the ith heat
y	=	dependent variable
$\overline{y}$	=	average of the dependent variable
$\hat{y}$	=	estimate of the dependent variable
λ	=	V_B/V_w, ratio of between to within-heat variance
σ	=	stress or standard deviation
σ_T	=	total standard deviation
σ^*	=	estimate of standard deviation for a multiple heat data collection
σ^2	=	within-heat component of variance
ω^2	=	between-heat component of variance

1. INTRODUCTION

In 1976, a subcommittee from the MIL-HDBK-5 Coordination Committee was established to investigate the state of the art in creep and stress rupture data analysis procedures. The goal of this Elevated Temperature Task Group (ETTG) was to develop a new and more comprehensive guideline on creep and stress rupture data analysis for inclusion in Chapter 9 of MIL-HDBK-5.

After undertaking this task, it eventually became apparent that any guideline resulting from this activity could not be totally comprehensive, since the limited length of MIL-HDBK-5 guidelines precluded a comprehensive review of the state of the art that would completely support recommended analysis procedures. In view of this predicament, a compromise approach was taken. A relatively brief guideline which delineated the appropriate methods for creep and stress rupture analysis for MIL-HDBK-5 was prepared. This guideline, as approved at the 58th MIL-HDBK-5 Coordination Meeting is included in Appendix A. The guideline covers all major considerations, but it does not go into any great detail on the justification for such an approach, or on particular difficulties that one might encounter in special cases which would complicate the analysis.

The purpose of this reference document is to provide supplementary background information on creep and stress rupture analysis which will be useful in performing a data analysis according to the MIL-HDBK-5 guidelines. The document is subdivided into three sections: (1) design of experiments for the purpose of developing regression models, (2) required correlative information for use in evaluating elevated temperature property data, and (3) a comprehensive method of rupture data analysis with simplified models.

2. DESIGN OF EXPERIMENTS FOR THE PURPOSE OF DEVELOPING REGRESSION MODELS

There are hundreds, perhaps thousands, of curves published and used in industry today for the purpose of graphically displaying the mechanical properties of metals and their alloys. The vast majority of the curves that manufacturers use in designing everything from metal fasteners to aircraft

turbines are developed from data not specifically generated to produce mathematical models. The data, if excessive, raise costs; and if not sufficient, decrease reliability. Often creep and stress rupture data are both excessive and inefficient because the observations, although more than required, do not adequately cover the temperature-life matrix. In practice, mathematical models are derived and curves are drawn from those models. Product reliability depends on the accuracy of these curves. To develop precise and accurate models, a system of test planning for regression analysis is required. It is the purpose herein to present one simplified method of experimental design that will enable engineers to produce a more precise model at less cost. The production of the models is not covered in this section since it is discussed in detail in Section 4.

2.1 Planned Testing

Planned testing using experimental design techniques for selection of experiments is a logical approach to understanding the properties of an alloy, yet it is almost never done. The designer's success is directly related to the efficiency of the design curves and mathematical models developed from test data.

One method for planning testing is to develop a test layout in matrix form with the test temperatures listed in rows and lifetimes of interest tabulated in columns. Then, through testing, the blocks formed in the row-by-column matrix can be filled out. This approach ensures coverage of all the areas of interest.

Although there are other methods, this method is probably the simplest, oldest, and most generally used one; the most efficient and sophisticated method is the "Central Composite Design". Other methods are discussed in Reference 2-1.

Using the matrix format shown in Figure 2.1, this method involves the steps listed as follows:

1. Select and bracket the range of temperatures and time variables desired. Insert the temperatures selected in the extreme right-hand column.

2. From estimated log-log or Larson-Miller typical (average) life plots select and record the stress expected to produce the column life at the row temperature.
3. If no stress-temperature or time related interactions are expected, some of the experiments can be omitted, so long as no less than 20 experiments remain.
4. Omitted blocks must be selected randomly as follows:
   a. Omit blocks of experiments in sets equal to the number of temperatures in the model.
   b. In each set, omit one observation from each temperature level in the rows. (See the example given in Section 2.3.)
   c. Do not omit more than one value in any life (hours) columns.
5. For small designs do not omit any of the corner experiments (in the matrix).
6. If interactions are suspected or found, the entire matrix must be completed (no omitted blocks are allowed).
7. Perform the experiments in random order, mixing temperatures, machines, operators, heats, etc., to ensure that unaccounted for (nuisance) variables are randomized.
8. Small differences in the experimental life obtained and the estimated life are to be expected. Reset the stress levels and readjust the matrix if an experimental life is off by more than two columns from the estimate. All experimental results may be used in the final regression.

Before the test matrix, as shown in Figure 2.1, can be formed, the interval sizes must be selected, first for test temperatures, and then for desired lifetimes.

(a) Temperature - A range of temperatures is usually required. For example, if the test range is from 1000 F through 1500 F, the basic question is: should tests be performed at six levels (1000 F, 1100 F, 1200 F, 1300 F, 1400 F, 1500 F) or at three levels (1000 F, 1300 F, 1500 F)? The decision can be quite complicated and based on such considerations as:

(1) The expected spacing of the isothermal lines

(2) The likelihood of parallel or divergent isothermal lines

(3) Anticipated precipitation of secondary phases during the life ranges of interest.

HOURS

TEMP		3	6	10	15	32	56	100	180	320	560	1000	3000	5600		F	
T1																	
T2																	
T3																	
T4																	
T5																	
T6																	
T7																	
T8																	

FIGURE 2.1. MATRIX FORMAT

If reasonable estimates of anticipated isothermal lines can be constructed, this selection can be greatly simplified with very little risk. Starting with the lowest temperature, the next temperature line should be chosen such that at least one level of testing stress, on the log-log stress-life plot, will be common to both temperatures. This process should be repeated for each temperature line, ensuring like stress values for adjacent temperature levels.

(b) Life - A log life cycle should normally be divided into four equal intervals. For example, between 100 hours and 1000 hours, the divisions would be approximately 180, 320, and 560 hours on the log scale.

These divisions are far enough apart to insure a well defined curve and a minimum overlap of data. To convert from temperature and life desired to temperature and test stress requires some prior knowledge of the time-temperature-life relationship. If there is no prior knowledge, a series of "probe" tests must be made to locate the isothermal lines on a log-log plot. An example of an experimental design for the purpose of developing regression models is given in Section 9.3.6.8 of Appendix A.

2.2 Specification Data

Virtually all alloys are controlled and purchased to a material specification which provides a process control variable generally called the "spec point". Therefore, there will often be large quantities of data available from quality control data records at the "spec" condition. The data will contain many heats and will provide an excellent indication in regression equations of scatter. Therefore, in regression modeling, "spec" data are often the major source of the scatter or variability measurements. The slope measurements must come from the experimental design matrix.

"Spec" data can also be used to: (1) determine through analysis-of-variance techniques the fractions of the scatter due to heat-to-heat variations, etc., and (2) determine through distribution analysis if the data are normal, log normal, etc., and to determine, if the distribution is not normal, what is required to "normalize" it.

When no "spec" data are available, the scatter measurements are determined from the residuals of the regression model. However, they are mixed (confounded) with some small curve fitting error. Curve fitting usually increases scatter in stress rupture regressions by 5-15 percent. However, curve fitting errors can be much larger in some cases.

2.3 Heat-to-Heat Variation

A batch of an alloy is generally referred to as a heat. Batch variations in chemistry, heat treating, etc., can cause considerable variations in the mechanical properties of the alloy. This difference is referred to as the heat-to-heat component of variance as opposed to the within-heat component of variance. Heat-to-heat standard deviation is often 30-70 percent of the within-heat standard deviation although in some cases it may be much larger. The root sum square of the two components of variance produces a measure of scatter about the regression, that when added to the curve fitting error, gives the regression parameter called SEE (Standard Error of Estimate). It is this parameter which is used to fix the design minimums about the regression estimates (typical, or mean values). The SEE is rarely determined as defined above, rather it is a product of regression analysis.

Two methods are generally used to obtain the major components of variance, between (heat-to-heat) and within-heat. One method is described in Section 4.6 on Multiple Heat Data. A second method, based on Analysis-of-Variance (ANOVA) techniques, is much simpler and is described by an illustration. This method requires repeat observations, from several heats, at a common stress-temperature level (such as at the specification point).

Table 2.1 shows 19 times to stress rupture, in hours, for specification data associated with four heats, BJJK, BJJJ, BKLJ, and BLLD. Since a log-normal distribution of rupture lives will be assumed, base 10 logarithmic transformations must be made of these lifetimes before making subsequent analyses.

TABLE 2.1. STRESS RUPTURE TIMES TO FAILURE (HOURS)

Heat Label			
BJJK	BJJJ	BKLJ	BLLD
35.0	51.3	29.0	41.4
33.1	37.5	36.1	16.5
33.4	48.6	47.5	33.6
42.7	74.2		32.6
70.5			27.4
			26.4
			34.9

Table 2.2 shows standard computations (see Reference 2-2) that partition the total sum of squared deviations TSSD into two components: the between-heat component, BSSD, and the within-heat component, WSSD. These components are defined in algebraic terms, together with their associated computational formulas, as follows:

$$\mathrm{BSSD} = \sum_{i=1}^{h} n_i\left(\bar{x}_i - \bar{x}\right)^2 = \sum_{i=1}^{h} S_i^{\,2}/n_i - \left(\sum_{i=1}^{h} S_i\right)^2 \Bigg/ \sum_{i=1}^{h} n_i \quad ,$$

$$\mathrm{WSSD} = \sum_{i=1}^{h} \sum_{j=1}^{n_i} \left(x_{ij} - \bar{x}_i\right)^2 = \sum_{i=1}^{h} \mathrm{WSSD}_i \quad ,$$

and

$$\mathrm{TSSD} = \sum_{i=1}^{h} \sum_{j=1}^{n_i} \left(x_{ij} - \bar{x}\right)^2 = \sum_{i=1}^{h} SS_i - \left(\sum_{i=1}^{h} S_i\right)^2 \Bigg/ \sum_{i=1}^{h} n_i \quad .$$

From the algebraic expressions, it is seen that the TSSD consists of the sum of the squared differences between each observation x_{ij} (the jth observation in the ith heat) and the mean *taken over all observations*, $\bar{x}$.

In contrast, the WSSD is seen to consist of the sum of the squared differences between each observation, within a given heat, x_{ij}, and the mean for that heat, $\bar{x}_i$. These within-heat sums of squared deviations, WSSD_i are then summed over the h heats to obtain the final WSSD. The between-heat term, BSSD, is obtained by summing the squared differences between the mean of each heat $\bar{x}_i$ and the overall mean $\bar{x}$. These differences are weighted in accord with the number of observations in each heat n_i. An algebraic expansion of these expressions shows that

$$\mathrm{TSSD} = \mathrm{BSSD} + \mathrm{WSSD}$$

The right-hand side of the defining expressions gives the conventional computational formulas, where the symbol S_i denotes the sum of the observations in the ith heat, and SS_i denotes the sum of the squares of the observations in

TABLE 2.2. COMPUTATIONS FOR A COMPONENTS OF VARIANCE ANALYSIS FOR STRESS RUPTURE LIFETIME (LOG_{10})

Heat Code	Heat No. i	$\log_{10}$ (stress rupture life)					Sample Size n_i	Sample Sum S_i	Sum of Squares SS_i	S_i^2/n_i	$WSSD_i$	Sample Mean X_i	Sample Std. Dev. s_i
BJJK	1	1.544	1.520	1.524	1.630	1.848	5	8.066	13.090	13.013	0.077	1.613	0.139
BJJJ	2	1.710	1.574	1.687	1.870		4	6.841	11.745	11.700	0.045	1.710	0.122
BKLJ	3	1.462	1.558	1.677			3	4.697	7.377	7.354	0.023	1.566	0.108
BLLD	4	1.617	1.217	1.526	1.513	1.438	7	10.276	15.185	15.085	0.099	1.468	0.129
		1.422	1.543										
		Total					19	29.880	47.397	47.152	0.244	--	--

the ith heat. The computational formulas are used to reduce round-off errors in hand calculations.

Based on the totals shown in Table 2.2, it is seen that

$$BSSD = 47.152 - (29.880)^2/19 = 0.162$$

$$WSSD = 0.244$$

and

$$TSSD = 47.397 - (29.880)^2/19 = 0.407.$$

Table 2.3 shows an analysis of variance table that summarizes these results. The degrees of freedom f for the between-heat SSD, the within-heat SSD, and total SSD are given by $h-1$, $\sum_{i=1}^{h} n_i - h$ and $\sum_{i=1}^{h} n_i - 1$, respectively; and with $h = 4$ these expressions yield 3, 15, and 18 as shown in Table 2.3. The corresponding mean squares are given by SSD/f and are also shown in the table.

TABLE 2.3. ANALYSIS OF VARIANCE TABLE FOR STRESS RUPTURE LIFETIME (LOG_{10})

Source of Variance	Sum of Squared Deviations SSD	Degrees of Freedom f	Mean Square SSD/f
Differences Between Overall Average Lifetime and Average Lifetime for Each Heat	0.162	3	0.0540
Differences Between Average Lifetime for Each Heat and the Individual Lifetimes Within Each Heat	0.244	15	0.0163
Total	0.406	18	--

The components of variance are obtained from the mean squares as follows (see Reference 2.3). Let MSB and MSW denote the mean squares for the between-heat variation and the within-heat variation. Next, let ω^2 and σ^2 denote the components of variance for the between-heat and within-heat variations. The expected value of MSW is given by σ^2, so that an estimate of σ^2 is equal to 0.0163 for this example. The expected value of MSB is given by $\sigma^2 + \bar{n}\omega^2$, where $\bar{n}$ is an "average" number of observations per heat and is computed using

$$\bar{n} = \left[\sum_{i=1}^{h} n_i - \left(\sum_{i=1}^{h} n_i^2 / \sum_{i=1}^{h} n_i\right)\right] / (h-1)$$

which for this example becomes

$$\bar{n} = (19-(99/19))/3 = 4.60 \quad .$$

An estimate of ω^2 is then obtained from the relation: $0.0163 + 4.60\omega^2 = 0.0540$ and is found to be given by $\omega^2 = 0.0082$. Taking square roots then shows that the standard deviations of the between-heat and within-heat components of variance are estimated by $\omega = 0.091$ and $\sigma = 0.128$, approximately.

Because a single measurement of lifetime is likely to be affected by heat-to-heat variation, and by the variations within a heat, an estimate of the standard deviation for a single subsequent measurement, σ^*, can be obtained using the expression:

$$\sigma^* = \sqrt{\omega^2 + \sigma^2} = \sqrt{0.0082 + 0.0163} = 0.157 \quad .$$

The ratio of the heat-to-heat component of variance to the within-heat component of variance is given by $\omega^2/\sigma^2 = 0.0082/0.0163 = 0.50$. Thus, for this example, the heat-to-heat variation is approximately equal to 50 percent of the within-heat variation. As a rule of thumb, the following requirements for the number of heats have been established for creep and stress rupture data analysis in MIL-HDBK-5:

(1) When the heat-to-heat component of variance is less than 25 percent of the within-heat variance, use at least two heats equally for the sample sources.

(2) When the heat-to-heat component of variance is between 25-65 percent of the within-heat variance, use at least three heats equally.

(3) When the heat-to-heat component of variance is greater than 65 percent of the within-heat variance, use at least five heats equally.

For the sample set of data examined above in Tables 2.2 and 2.3, data on at least three heats of material would be recommended by MIL-HDBK-5 for analysis purposes.

When regression models are developed from data that were not taken from an experimental design, the heats are rarely chosen randomly. Therefore, unless there are large quantities of data in all areas of the regression matrix, this imbalance of heat sample sizes must be accounted for. The manner in which this is done will not be covered here; one method is described in Reference 2.4 and is also briefly reviewed in Appendix B.

2.4 Summary:

To design the experiments necessary to produce reliable stress rupture or creep curves and to develop realistic mathematical regression equations from those experiments, the following factors must be established:

1) Temperature: Choose the specific isothermal levels from the temperature range of interest.
2) Number of Observations: Determine how many observations are to be made, and at what stress levels, for each isothermal temperature level.
3) Number of heats: Determine how many heats should be selected and randomize the heats equally throughout the test matrix.

3. CORRELATIVE INFORMATION FOR USE IN EVALUATING ELEVATED TEMPERATURE PROPERTY DATA FOR METALLIC MATERIALS

3.1 Need for Correlative Information

The properties of metallic materials operating at elevated temperatures are influenced strongly by processing variables. Every phase of processing, from solidification of the alloy from the molten state to the final heat treatment, affects the magnitude of individual property values and the statistical variation of those values representing both individual pieces and tonnage quantities of those pieces. Paralleling this fact is the complexity of the interplay at elevated temperatures among purely metallurgical phenomena; the mechanical aspects of stress, short-time strain, long-time strain; and fracture. The properties of metallic materials at room temperature vary much less as a function of processing variables.

Data for elevated temperature properties of alloys frequently are not generated for the purpose of providing a basis for allowables to be used in design. Variables frequently are not selected and controlled to provide optimum data, but are those encountered in evaluating materials for other purposes. Thus, it is necessary to evaluate not only the property data, but correlative information as well, to assess the suitability and reliability of those data for use in establishing design allowables for general use.

Frequently when data are generated for other than pure "data generation" purposes there is a significant amount of correlative information available, but it is not recorded or is recorded incompletely. In some instances, of course, information is simply lacking due to the narrow objectives of the specific testing concerned. The originator of the test can be assisted by having available a list of the various identifying information which are (a) necessary and (b) desirable for potentially extending the usefulness of the data generated beyond its original intent. The intent of the sections which follow is to establish a list of correlative identifying information for use by anyone generating elevated temperature property information for metallic materials.

3.2 Detailed Uses for Data Generated and Correlative Information

Elevated temperature property data are used for many individual purposes, some of which are broader and of greater overall significance than others. Nevertheless, all elevated temperature data must be supported by a certain amount of correlative information. The amount of required correlative information is a function of how the data are used.

Examples of different data uses are:

a. Accumulation, analysis, and presentation of data for MIL-HDBK-5.

b. Evaluation of new materials.

c. Evaluation of field-retired parts.

d. Evaluation and investigation of failures of parts.

MIL-HDBK-5 data analyses normally concern materials which have been in production in relatively large quantities over a period of some years, which have been processed by different methods suiting end requirements, which are relatively well established, and for which there exists a substantial body of information. Due to the far-reaching significance and use of property values in MIL-HDBK-5, considerable care must be used to assure that the data being evaluated are indeed representative of production material, and a great part of that assurance comes from correlative information relating to those data.

For new materials, a relatively modest amount of experience will have been accumulated since smaller quantities will have been produced and processing methods will probably still be in a state of development. In this case, documentation of the processing is vital not only for immediate purposes, but to permit future consideration of the data (perhaps for inclusion in MIL-HDBK-5) and to determine whether those data are representative of material currently being produced.

For field retired parts and failed parts, less information is normally available. Failure analysis at times can be quite straightforward [defect not detected by nondestructive testing (NDT)], but at other times (such as abnormally low short-cycle fatigue properties) it can involve investigation to obtain complete processing information.

3.3. Nature of Correlative Information to be Provided with Property Data

Collection of correlative information: The first step is to collect all information immediately at hand without additional effort or cost, regardless of the kind of information. Reports or documentation from the producer, NDT inspection reports, analytical data from chemical laboratories, and specification data are examples of information which may already be at hand.

Categories of information to be supplied: The second step consists of determining the identity of information falling into the following categories:

(1) Always to be supplied.

(2) Supplied if obtaining it is feasible and not excessively costly or time consuming.

(3) Supply only if already on hand.

3.4 Information Regarding the Material Product

General identifying information regarding the material product should include:

Identity of producer of

(2) a. Intermediate material used in producing final product.

(1) b. Final product tested.

Information relating to all products

(1) a. Identity of metal or alloy

(1) b. Procurement specification to which it was produced

(3) c. Purchase order number to which the final product was procured

d. Usual pedigree information:

(1) Chemical analysis determined at the producing mill

(2) Check analysis

(1) Heat number, if applicable

(3) Ingot number, if applicable

(2) Mill test report (frequently contains supporting supplementary data)

(2) Tensile property and fracture toughness data generated at mill or elsewhere to assure that product conformed to specification requirements

(3) Hardness

(3) Crack propagation data (da/dN) if available

e. Quality aspects

(3) Cleanliness rating data (AMS 2300, 2301)

(3) Macroetch rating

(2) Ultrasonic class to which product conforms if required (AA, A, B)

(2) Radiographic rating, as applicable

(2) Eddy current rating, as applicable

f. Heat treatment

(1) Conducted by whom (producing mill, forge shop, user, etc.)

(1) Final heat treatment conducted on as-produced stock, rough-machined stock, or finish-machined part

(2) In air, vacuum, inert gas, cracked gas, etc.

(2) Statement of time, temperature, quenching, tempering, aging, etc., if not adequately covered elsewhere

(1) Sequence and nature of mechanical working if such is associated with achieving desired heat treat condition

Information relating to specific products

(1) Results of any microstructural studies made on the specific product tested, or on specimens from it

Aspects of melting and casting process used in producing ingot from which final product is made (usually applies only to iron-and titanium-base alloys)

(1) a. Melting process

(2) b. Number of remelt cycles, as applicable, especially if number is abnormal

(3) c. Special aspects of melting and casting, if applicable (all high purity melting stock; inert gas used over ESR slag layer, etc.)

(3) d. Ingot size, especially if abnormal

(3) e. Conditioning of ingot surface, especially if abnormal

Forgings

(2) a. Hammer, hydraulic press, HERF machine, ring

(1) b. Forging practice: Closed die, pot forging, hand (Smith) forging, ring rolling, loose mandrel ring forging

(1) c. Forging stock used: Forging billet, forging ingot, powder metallurgy preform

(3) d. Size of forging stock used

e. Out-of-the-ordinary aspects, as applicable:

(1) Low forging temperatures in HERF product (initial and final)

(2) Creep forging in superalloy dies

(2) Ausforming time-temperature sequence

(3) Time-temperature-percent reduction of powder metallurgy preforms

Extrusions

(3) a. Size of press used

(1) b. Extrusion stock used: Billet, ingot, powder metallurgy preform

(3) c. Size of extrusion stock used

(2) d. Out-of-the-ordinary aspects, e.g., low or high reduction ratios

Impact extrusion (impact forging, cold forging)

(3) a. Kind and size of press used (mechanical, hydraulic)

(2) b. Type of extrusion: forward, backward, forward and backward

(3) c. Shape and design of preform

(1) d. Stock used in fabricating preform

(3) e. Temperature of preform

Castings

(1) a. Process: green sand, baked sand, shell mold, plaster, investment, etc.

(3) b. Type of cores used

(3) c. When available, location of gates and risers (as they can affect properties in their vicinity)

(1) d. Whether casting was repair welded before final heat treatment (very common), and if so where and how

(1) b. If specimen is from a casting, does it have as-cast surfaces or was it machined all over

(1) c. If specimen represents a cast product, was the specimen cast separately

(1) d. Does the specimen retain some surfaces from the original product, e.g., as forged and as-heat treated surface of a die forging

(3) e. Sequence followed in rough machining, finish machining, and heat treatment

(1) Sequence and nature of mechanical working if such is associated with achieving desired heat treat condition

Information relating to specific products

(1) Results of any microstructural studies made on the specific product tested, or on specimens from it

Aspects of melting and casting process used in producing ingot from which final product is made (usually applies only to iron-and titanium-base alloys)

(1) a. Melting process

(2) b. Number of remelt cycles, as applicable, especially if number is abnormal

(3) c. Special aspects of melting and casting, if applicable (all high purity melting stock; inert gas used over ESR slag layer, etc.)

(3) d. Ingot size. especially if abnormal

(3) e. Conditioning of ingot surface, especially if abnormal

(1) f. Heat treatment practice followed, if such was conducted on the machining blank or on the finished specimen, thickness of blank heat treated

(3) g. Heat treat oxide left on or removed; if removed, how was it accomplished

(1) h. Identity of any surface-finishing processes used, and surfaces on specimen so affected: Dissolving

surface-damaged layer in beryllium; peening (specification used and practice followed)

(2) i. NDT of finished specimen, including radiographic quality of welds

Specimen proper

(1) a. Basic design and dimensional tolerances

(1) b. Location of specimen in product, including orientation with respect to grain flow and with respect to weld direction

(1) c. Notch orientation with respect to L, LT, and ST directions

(2) d. Precracking practice

(1) e. Sequence of precracking and heat treatment.

Powder metallurgy end product

(2) a. Powder production method

(3) b. Powder size and, if applicable, shape characteristics

(3) c. Pressing method (axial ram-type, isostatic, hot or cold, pressure, time)

(3) d. Sintering temperature, time, and atmosphere

(3) e. Special aspects, e.g., use of activators with powder

(3) f. Pressing and sintering practice: pressures, atmospheres, density as pressed, sintering time-temperature-atmosphere, density after sintering

Weldment

(1) a. Process: TIG, MIG, EB, coated-electrode, etc.

(2) b. Weld face preparation

(1) c. Filler metal used: kind, size, procurement spec.

(2) d. Practice: number of passes, preheat, interpass temperature, postheat, amperage, voltage, type current, limitation on joules per inch, etc.

(1) e. Heat treat condition: as welded, reheat treated, etc.

3.5 Information Regarding the Test Specimens

General identifying information regarding the test specimens should include:

Identity of stock from which specimens were machined

(1) a. Virgin product, e.g., plate as produced by the mill

(2) b. Product subject to prior testing, e.g., tests on a field-retired part, or on a statically tested forging

Fabrication

(1) a. If applicable, identity of any nontraditional (non-mechanical) machining process used, such as EDM (electrical discharge machining) or ECG (electrochemical grinding). Some of these processes, such as EDM, may alter test surfaces significantly

(1) b. If specimen is from a casting, does it have as-cast surfaces or was it machined all over

(1) c. If specimen represents a cast product, was the specimen cast separately

(1) d. Does the specimen retain some surfaces from the original product, e.g., as forged and as-heat treated surface of a die forging

(3) e. Sequence followed in rough machining, finish machining, and heat treatment

(1) f. Heat treatment practice followed, if such was conducted on the machining blank or on the finished specimen, thickness of blank heat treated

(3) g. Heat treat oxide left on or removed; if removed, how was it accomplished

(1) h. Identity of any surface-finishing processes used, and surfaces on specimen so affected: Dissolving surface-damaged layer in beryllium; peening (specification used and practice followed)

(2) i. NDT of finished specimen, including radiographic quality of welds

Specimen proper

(1) a. Basic design and dimensional tolerances

(1) b. Location of specimen in product, including orientation with respect to grain flow and with respect to weld direction

(1) c. Notch orientation with respect to L, LT, and ST directions

(2) d. Precracking practice

(1) e. Sequence of precracking and heat treatment.

3.6 Information Regarding the Testing

(1) Basic type of test

(2) If available, specification for test procedures followed, e.g., ASTM E139-70 for creep-rupture testing

(1) Details of testing if not covered completely by a specification

(2) Identity and capacity of equipment used for loading specimen

(3) Identity of strain-measuring instrumentation

(3) Identity of recording instrumentation

(1) Environment of specimen during testing: air, percent relative humidity, dew point if low humidity, salt fog, inert gas, torr vacuum, etc.

(2) If fracture faces evaluated, method used.

(1) Testing conducted by:

a. Name of company, laboratory, or other applicable organization

b. Name of individual personally conducting the test

(1) Correlation and analysis of data conducted by: (specify)

(3) Authorization and funding by: (specify)

4. A COMPREHENSIVE METHOD OF CREEP AND STRESS RUPTURE DATA ANALYSIS WITH SIMPLIFIED MODELS

4.1 Introduction

Until the dream of a universal creep and stress rupture equation is achieved, (that includes among its variations all the parametric and other relationships found useful in the past), it is necessary to use less general equations in curve fitting. The method described herein uses standard parametric equations as starting points and adjusts the best of these to correct for inadequate fit to the data. Depending on the type of lack-of-fit observed, the adjustments are made by the analyst, using metallurgical information where applicable.

The proposed method requires the analyst to consider the material as an inherent part of the analysis and to apply his mathematical skills where needed to closely approximate in a rational way the behavior interpreted from the data or learned from other sources. On the other hand, the method has been designed to provide the analyst with mathematical tools for obtaining good results from typical engineering data, permitting more efficient design.

Access to a computer is required and a degree of automation is strongly recommended.

4.2 Objectives of Data Analysis

The first objective of a creep or stress rupture analysis is to find the underlying relationship between stress, temperature, and life, modified if necessary by inclusion of the effects of such auxiliary variables as specimen geometry, grain size, coating, etc. In a creep or stress rupture analysis, the logical dependent variable is the logarithm (or log) of life; this variable is approximately normally distributed with uniform variance and has the

highest variability.* Least squares regression minimizes the sum of the squares of the differences between observed and predicted logarithms of life. The analysis must minimize fitting error to ensure that predicted lives lie in the central part of the envelope of the observed data over the range of test conditions.

Once the equation is obtained, it may be used to predict the stress to obtain a given life at a given temperature, or to predict the temperature to obtain a given life at a given stress. Either prediction can include appropriate statistical limits.

A second objective is to present the results in a form having high utility to the users—especially those in design engineering. In addition to graphs of the relationship, the results may be presented in the form of equations allowing predictions of probable creep or stress rupture life at any condition of design interest.

Any limitations on the range of applicability of published equations can be indicated on associated graphs by terminating the curves at the limit of applicability. Equations made available by publication or storage in computer programs require a definition of circumscribing limits, for example, stress and temperature limits on reliable extrapolation.

A third objective is to use the results of the analysis to improve the data mix to be obtained in the same or subsequent experiments. Principal interest lies in identifying test conditions which will optimally determine curve shape. Test conditions most frequently lacking in a creep or stress rupture analysis are those resulting in long lives. Other test conditions can frequently be found where cost-effective contributions to curve shape determination can be made.

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*The use of the logarithm of life as the dependent variable is supported by considering data at one stress and temperature, such as quality control data at a specification point. Life is the only variable. Statistical results from such data can readily be used in conjunction with regression analysis using the logarithm of life as the dependent variable for data at other test conditions.

4.3 Guiding Principles

In general, an attempt should be made to use all available data in the analysis, including those from different heats, having different grain size, section size, etc. However, all the data should not normally be made to fit a single curve. Effectively, a separate time-stress-temperature curve for each significantly represented heat and each level of each auxiliary variable should be obtained through the use of additional regression variables (terms in the equation). Regression variables for such physical characteristics as grain size and section size should be added to the equation when they are known quantitatively and when preliminary plots show they have significant effects on creep or stress rupture life. If the effect is not the same at all stresses and temperatures, cross-product terms of these regression variables with stress or temperature can also be added. The procedure that should be used for handling multiple heats is described in Section 4.6. Certain variables such as heat treatment or large changes in chemistry may affect curve shape so drastically that it will not be realistic to analyze data with different values of these variables in a single analysis, even when they are in the range allowed by the controlling specification.

Initial screening analyses should be made using standard parametric equations. These equations recommended in the MIL-HDBK-5 guidelines are known by their originator's name: the Larson-Miller, Dorn, Manson-Succop, and Manson-Haferd equations. An unnamed equation with linear terms in stress, reciprocal stress, and reciprocal temperature, and cross-product terms in stress with temperature and reciprocal temperature has also been found to be useful in some cases. The parametric equations which are recommended are cubic functions in the logarithm of stress or, in the case of Larson-Miller, a cubic function in stress.

When one of these equations fits the data adequately by the criteria described in Section 4.5, the analysis can be considered complete. For some alloys, there are regions of stress and temperature where none of the parametric equations will be adequate. In these instances, the most promising equation should be modified to obtain good fit.

A few words are in order about the recommendation to use a cubic log stress function in the screening equations. A cubic equation implies a sigmoidal shape on a logarithmic plot, and is usually concave downward at high stress and concave upward at low stress (the coefficient of the cubic term is negative). The high-stress curvature is needed in order for the equation to approach the ultimate tensile strength at reasonably short life and is generally accepted as valid. The curvature at the low stress end is questioned; some highly qualified observers favor an asymptotically linear curve in this region or even one with concave-downward curvature. With typical aircraft engine alloys the cubic representation is generally successful.

The inflection point of cubic analyses, where the curve is nearly straight, occurs near the low-stress limit of typical sets of data. Such data will not provide sufficient information to allow selection between the cubic and other curve shapes essentially linear in the low-stress region. This lack of definition is especially severe if long-time data are excluded from the analysis in order to test extrapolation capability. Relatively few data sets clearly show the existence of an inflection point; most show only a tendency toward linearity in this region. The use of the cubic may provide unconservative predictions with extrapolations greater than a decade toward longer creep or stress rupture lives if the linear or concave-downward assumption is really correct and data are simply not available to demonstrate this trend. The cubic is normally acceptable for moderate extrapolation, however.

One method that can be used to improve the fit is to use a double cubic, or cubic spline fit, in log stress to allow different curvatures at high and low stresses. The selection of the location of the knot (or value of log stress where the two cubic equations join) is arbitrary, but not generally cricical. Frequently the log stress value from the mid-range of the data is effective. When a cubic spline fit is used, both cubic coefficients for the equation of the line should be negative.

Analysis of data covering a limited range of stress will occasionally determine a positive coefficient of the cubic term in the parametric equations. This effect can be caused by unusual experimental error in a few data points strategically located to affect curve shape, and is not considered to represent a true relationship between stress and life. The following steps

should force the coefficient of the cubic term to be negative. First, eliminate the second order term in log stress. Second, by inspection of a log-log plot of the data, select a reasonable stress for the inflection of the curves. Divide all stresses by the value of the inflection stress before taking logarithms. This will force the inflection to the selected stress and allow the data to determine the curve shape under this restraint.

If insufficient high-stress data exist to determine curve shape, ultimate tensile strength data may be entered as rupture data at a short time, such as 0.01 hour, to control extrapolation in short times. Since the curve is extremely flat in the region of ultimate strength, the time assigned to these entries is not critical.

Runouts, or tests not run to completion for any reason, can be used in two ways. The first is to treat the data as censored and use a computer program which calculates the maximum likelihood relationship between the variables. This method is rather cumbersome, however, and tends to underestimate the standard error. Therefore it should generally be avoided. The method found more suitable, though less rigorous, is to include in a second regression only those runout data that lie above the curve calculated for completed tests (valid failures). It is reasonable to assume that runouts below the curve for valid failures contribute little to the determination of curve shape, while the inclusion as failures of those above the curve will tend to raise the curve where they occur. While not proven, this approach is considered to give conservative results on the average.

The use of higher order polynomials should be avoided because of their typically very poor extrapolative characteristics. When interaction terms between auxiliary variables are needed, these should generally be formed by multiplying the auxiliary variable by one of the stress or temperature terms already in the equation, thereby keeping the curve shape as simple as possible.

The usual temperature term is either linear, or the reciprocal of absolute temperature. If it is necessary to improve the fit with temperature, other powers than 1 or -1 can be used to give a simple curve shape with lower error. If the optimum exponent appears to be close to zero, the logarithm of

temperature is suggested since very small exponents cause roundoff errors in computer calculations.

4.4 Regression Analysis

Regression analysis can be performed using centered data to avoid roundoff error. To do this, the equation fit by the computer should be of the form

$$y_j - y = \sum_{i=1}^{k} b_i (X_{ij} - \bar{X}_i)$$

for k dependent variables, where

$\bar{y}$ and $\bar{X}_i$ are the averages of these variables,
j is the index of a particular life datum and
y_j is the corresponding predicted value of log life.

For example, if y = log time, T = temperature, and X = log stress, the cubic Larson-Miller equation for a single heat can be written

$$y - \bar{y} = b_1 \left(\frac{1}{T} - \overline{\frac{1}{T}}\right) + b_2 \left(\frac{X}{T} - \overline{\frac{X}{T}}\right) + b_3 \left(\frac{X^2}{T} - \overline{\frac{X^2}{T}}\right) + b_4 \left(\frac{X^3}{T} - \overline{\frac{X^3}{T}}\right) \tag{4.1}$$

or

$$y - \bar{y} = b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 \quad ,$$

where X = log stress and the subscripted X's have an average value of zero. The b_is are the regression coefficients. The sum of squares of the deviations (or sum of squares error) of observed y from predicted y is

$$SSE = \Sigma \, (y - \bar{y} - b_1 X_1 - b_2 X_2 - b_3 X_3 - b_4 X_4)^2 \quad . \tag{4.2}$$

The solution for the values of b_i can be obtained by setting the derivative of the last equation with respect to each b_i equal to zero, and

obtaining a set of equations called the normal equations. For the example given in Equation 4.1 these are:

$$\Sigma\ [X_1(y-\bar{y})] = b_1\ \Sigma\ X_1^2 + b_2\ \Sigma\ X_1X_2 + b_3\ \Sigma\ X_1X_3 + b_4\ \Sigma\ X_1X_4$$

$$\Sigma\ [X_2(y-\bar{y})] = b_1\ \Sigma\ X_2X_1 + b_2\ \Sigma\ X_2^2 + b_3\ \Sigma\ X_2X_3 + b_4\ \Sigma\ X_2X_4 \qquad (4.3)$$

$$\Sigma\ [X_3(y-\bar{y})] = b_1 = \Sigma\ X_3X_1 + b_2\ \Sigma\ X_3X_2 + b_3\ \Sigma\ X_3^2 + b_4\ \Sigma\ X_3X_4$$

$$\Sigma\ [X_4(y-\bar{y})] = b_1\ \Sigma\ X_4X_1 + b_2\ \Sigma\ X_4X_2 + b_3\ \Sigma\ X_4X_3 + b_4\ \Sigma\ X_4^2$$

The summations on the right side of these equations form the X matrix which can be inverted to provide a solution for the b_i's; the solution minimizes the SSE, independent of the form of the distribution of error in y.

A more accurate solution can be obtained if, before inversion, the X matrix is first converted to the correlation matrix. In the correlation matrix all entries on the principal diagonal are 1, and the off-diagonal entries are between -1 and +1. the inverse of the correlation matrix can be converted to the inverse of the X matrix without significant loss of accuracy.

With this approach and occasional rewriting of variables to reduce correlation among the "independent" variables, it is normally not necessary to use double precision or complete orthogonalization of the independent variables to obtain good results.

It is generally not possible to identify suspect data until at least a preliminary curve is obtained, either graphically or by regression. Any data unusually distant from this curve, called outliers, are suspect. If review of the original test record indicates any serious question as to their validity, the data should be removed from further analysis.

Data may also appear to be outliers because of the selection of the wrong model for the analysis. However, if unusual deviations occur for several customary curve fits, the data may be deleted even if no experimental cause is found. This is a judgment decision. Although there is a certain probability of valid outliers, inclusion of outliers can cause the prediction

for the material to be in error. The reasons for any deletions should be made a part of the record of the analysis.

4.5 Evaluation of Fit

Frequently, the size of the standard error of the regression is adequate to determine the suitability of a regression model. However, the equation with the smallest standard error may fit the data poorly in one important region and compensates for this by unusually good fit elsewhere. A detailed study of the residuals (observed minus predicted log life) will be helpful to evaluate whether an adequate fit has been obtained.

A simple plot against the predicted lives is one method of evaluating residuals. Any trends upward or downward in the residuals, particularly at long lives, indicate an equation with poor extrapolation potential. This is the most frequent reason for choosing an equation with a slightly higher standard error over that with the lowest.

A table of the average deviation in each of several ranges of predicted life is another tool for evaluating residuals. Five cells are suggested at each temperature, each cell covering a fifth of the total range in predicted log life. The observed pattern of plus and minus deviations can suggest changes to the equation which will increase randomness. A computer program can easily be modified to print such a table as part of the analysis output; this can improve efficiency by avoiding plot routines for every trial.

An extension of a plot of the predictions, particularly to longer lives than those measured, will allow engineering judgment of the suitability of the equation for extrapolation. Typical problems that can be uncovered by this technique are crossing of isotherms and radical changes in curve slope outside the data range.

A good standard error is 0.10 in common logarithms of life; typical values usually range from 0.15 to 0.30. Occasionally standard errors as low as 0.05 can be obtained, particularly with small amounts of data and when complex equations are used. When this happens, it is likely that some of the error of the measurements is being fit by the equation. Extrapolation is usually unreliable in such cases.

The R^2 static, or coefficient of multiple determination is also useful in evaluating the quality of a regression fit. Lipson and Sheth (2-2) provide a clear definition of this statistic and its uses.

4.6 Multiple Heat Data

Draper and Smith (4-1) recommend the use of dummy variables to account for differences between subsets (e.g., heats) of data. This method requires adding a variable to the equation for each heat except one, increases the size of the matrix to be inverted, and thereby increases the chance of serious computer round-off errors.

Another approach is to use tensile or short-time stress rupture data as measures of heat variability by including functions of one or both as independent variables. This approach is particularly useful when predictions are to be made for a particular heat for which such data are available. Heat separation is then based entirely on the tensile or short-time stress rupture data used in the analysis; optimum heat separation in the sense of minimum within-heat variance is not likely to be obtained. Tolerance intervals for unknown heats are also likely to be in error. (Tolerance interval predictions are frequently needed for production runs using many heats.)

The method of heat separation recommended here, though equivalent to the dummy variable method, avoids the difficulties inherent in the two methods above. The data for each variable should be centered about the mean value for each heat. Equation 4.1 is replaced by

$$y_{hj}-\bar{y}_h = \sum_{i=1}^{k} b_i(X_{ij}-\bar{X}_{ih}) \tag{4.4}$$

where h is the index of the heat to which it belongs, and y_{hj} is the predicted value of log life for a specific heat. The heat separation results are identical to those obtained with the use of dummy variables, but are more precisely and efficiently determined. Including tensile data as very short-time stress rupture data has only a minor effect on determining the differences between heats since all data are used for this purpose.

When the method of heat separation is used, the sum of squares minimized by regression is based on the differences between observed values and the predicted values of log life for each heat. Variability between heats is accounted for, and an average constant for the material is calculated, by additional operations (described below) on the values of

$$c_h = \overline{y}_h - \sum_{i=1}^{k} (b_i \overline{X}_{ih}) \quad ,$$

the heat constants of the equation.

The dummy variables method and the heat separation method both assume parallel curves for each heat, that is, a constant ratio of lives between any two heats, at all test conditions. This is approximately true in many sets of creep or stress rupture data, although some lack of parallelism can be observed.

A statistical test of the significance of lack of parallelism can be applied. The residuals from each heat, which always sum to zero, may be fit to a linear equation against any independent variable, or against the predicted logarithms of life. The coefficients of such equations are generally not significantly different from zero, indicating that the data do not disprove the assumption of parallelism. In other words, apparent lack of parallelism, at least in part, is due to random scatter in the data.

Even if the true relationships for the various heats are not represented by exactly parallel curves, the average curve determined by heat separation is still a valid prediction for average material if the heats analyzed are representative of heats in the total population of heats. The assumption of parallelism will increase the within-heat sum of squares where the curves are not parallel. The tolerance interval will be more conservative and should not lead to over-optimistic predictions for design. Multiple-heat analyses using heat separation techniques usually give within-heat standard errors in the range of 0.10 to 0.30, the same as analyses for single heats.

Heat separation allows each heat tested at more than one condition to contribute to curve shape. Heats with only one test condition, such as specification point data, contribute only to the determination of the average constant and the between-heat variance for the material. When all data are

analyzed together without regard to heat, specification point data can significantly distort curve shape.

The analysis by heat separation techniques gives an equation with a separate constant term for each heat and a set of common coefficients. The relationship between the constant terms represents the relationship between the logarithm of life among the heats at any test condition. When the heats included in the analysis were not preselected for any particular characteristic, the constants may be considered to be a random sample from the population of all heats, and are assumed to have a normal distribution. When sufficient heats are present, a good measure of between-heat variance may be obtained.

The next step is to determine the constant for the material. It obviously should be based on some average of the constants for the individual heats. One way to do this is a simple arithmetic average of the heat constants. This is correct only when the between-heat variability is much larger than the within-heat variability. The second approach is to weight each heat constant by the number of data points in that heat. This procedure is correct only when the between-heat variability is much smaller than the within-heat variability. The heat separation method includes these two approaches as extreme cases and also correctly treats the more usual case where the between- and within-heat variabilities are of the same order of magnitude. Logically consistent between-heat variance is calculated at the same time as the average equation (material) constant.

Mandel and Paule (2-1) studied the situation where several laboratories measure a given property, but do not each make the same number of measurements. A mathematically simple iterative procedure was reported for estimating the "best" single value to represent the property, and of the between-laboratory variance. Creep or stress rupture data can be made to fit Mandel and Paule's model if "heats" (or other subsets of data) are substituted for "laboratories," "heat constants" for the "average property value from each laboratory," and the "square of the standard error from the regression" for the "pooled within-laboratory variance." A short paraphrase of this analysis is presented in Appendix B. Applied to creep or stress rupture data, the method is simple, iterates quickly, and gives results which stand up well to engineering inspection.

A batch program in a version of FORTRAN is recommended when data are retrieved from a data storage system or large amounts of data are supplied on punched cards. Several standard analysis packages are available. These include Statistical Analysis System (SAS) and Biomedical Computer Programs (BMDP).

It is recommended that a standard version of a batch program perform the analysis of the screening equations described previously. The batch program should be capable of operating on these screening equations with or without additional terms in auxiliary variables, or even with completely non-standard equations. The minimum output should include the standard error of estimate, the coefficients with their standard errors, and a table of predicted average lives for each heat and the average material curve. Some analysts may also wish to define a lower level tolerance bound on the average curve. Two plots of the data (coded by heat) should be generated, one with curves for all equations on isothermal plots and another with all isotherms on a single plot for the equation with the smallest standard error. Isothermal deviation plots should also be produced for this equation. The table of average deviations for five cells of predicted lives for each temperature can be printed for each equation. These recommendations reflect experience in obtaining the minimum necessary information in a labor-efficient manner without overwhelming the analyst with both paper and information.

4.8 An Example Analysis

A collection of 304 stainless steel data, reported by the National Research Institute for Metals in Japan, was obtained. Six heats of hot extruded material were represented in the collection of data. One of these heats, AAA, showed on preliminary plots and individual heat regression analyses to vary systematically from the other five heats and was not used in the analysis. Except for its unusually low boron content, no metallurgical reason was found for this difference.

One datum from heat AAB (1127.5 hours, 700 C, 8 kg/mm^2) was also discarded since it appeared as an outlier in both the individual heat analysis

and in comparison with the data at 700 C from the other heats. The remaining 78 points from five heats were used in the analysis.

Separate analyses of each heat using a Larson-Miller equation with a cubic function of the logarithm of stress had a pooled standard error of 0.092, an adequate measure of experimental error. Only one of these individual heat analyses resulted in an equation with acceptable extrapolative capability.

A plot of the data shows that the logarithm of time is approximately linearly related to the logarithm of stress, which may account for the unacceptable extrapolation of the cubic fits. The plot also shows a systematic variation in the relationship between heats. Although this indicates that the individual heat curves are not parallel, the assumption of parallelism was retained since it was expected to produce a reasonable curve for average material.

The first step taken to arrive at the final equation was to use a Larson-Miller equation that was linear in the logarithm of stress. The temperature exponent was adjusted from -1 to -0.1. Finally an exponential term of the form exp $[A \cdot (T-1202)^2]/\sigma$, was added with T in degrees F and σ in MPa. The within-heat standard error of the final equation is 0.127. The square root of the total variance is 0.175.

The final within-heat standard error is higher than the pooled standard error of the individual heat analyses (0.092) largely because of nonparallelism of the five heats tested. Two heats, AAE and AAF, fell above the average curve except at low stress and high temperature. The other three heats behaved generally in the opposite manner.

Figure 4.1 shows the data plotted as solid symbols for heats AAE and AAF and open symbols for the other heats. The curve for average material is also shown, extrapolated to 10^5 hrs. Its equation is

$$\log t = b_o + b_1/T^{0.1} + b_2 \log \sigma/T^{0.1} + b_3 \cdot \exp\left[A \cdot (T-1202)^2\right] /\sigma$$

where T is degrees F and σ is Mpa.

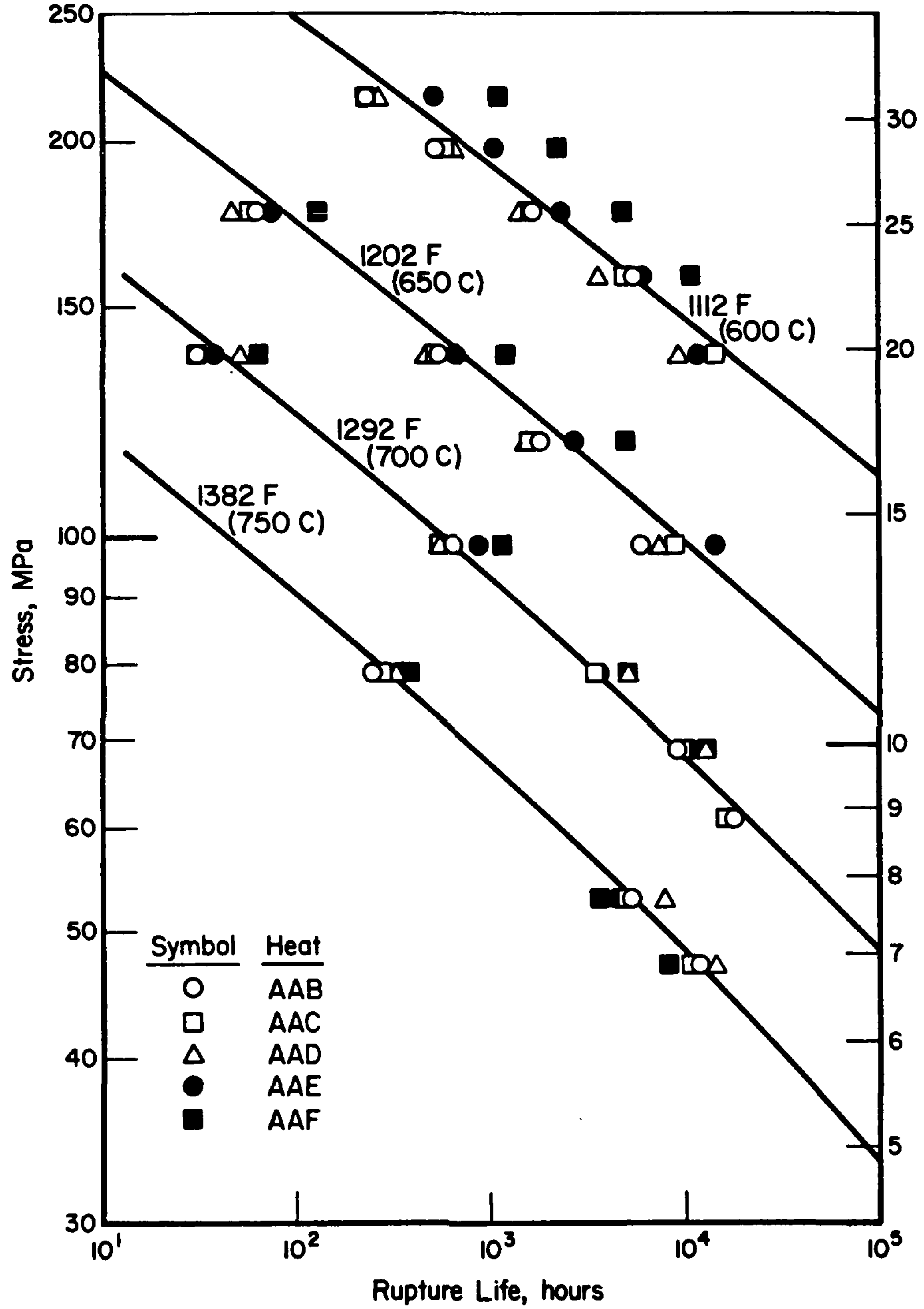

FIGURE 4.1. OBSERVED AND PREDICTED STRESS RUPTURE LIVES FOR NRIM 304 STAINLESS STEEL

A better analysis could have been performed on the two groups of heats separately, with considerably less lack of parallelism in each case, but this was not done because two or three heats are insufficient to accurately determine between-heat variance, and because there was no way to describe the subpopulation of heats to which each analysis would belong.

The exponential term in the equation represents an effect centered at 1202 F (650 C). This temperature was determined from the data upon iteration. It may be coincidence that 1202 F is also the center of the temperature range in which carbide precipitation and intergranular corrosion occur in this material, although the negative exponential term may be reflecting such a phenomena.

The statistical significance of the variation between heats under the assumption of parallelism is very high. The ratio of between-heat and within-heat mean squares is 11.6 with 4 and 70 degrees of freedom, and is significant with greater than 99.9 percent confidence. These data result in a somewhat unsatisfactory analysis in the absence of information about the differences between the two groups of heats (and the sixth heat which was not included in the analysis). The method of heat separation, however, can be expected to approach the true rupture relationship better than the simpler approach of analyzing all data as one population irrespective of heat, especially with only some of the heats being tested at some conditions of stress and temperature. The lack of parallelism did cause the standard error to increase, and consequently the prediction limits to expand, so as to compensate at least partially for this defect in the model.

4.9 Summary

The method of creep and stress rupture data analysis described in this section involves regression analysis of mathematical models, starting with standard parametric models and modifying them as needed to improve the fit and the ability to extrapolate reasonably. A method of handling multiple heat data is included in which the variability between specimens from a single heat and the variability between heats are treated separately. The method presented allows the determination of a single curve for the average properties of a material from data on representative heats of that material.

REFERENCES

2-1 Myers, R., Response Surface Methodology, Library of Congress Catalog No. 71-125611, 1976.

2-2 Hald, A., Statistical Theory With Engineering Applications, John Wiley & Sons, Inc., 1952 (See Chapter 9, Table 16.4, Table 16.5, p. 426).

2-3 Brownlee, K. A., Statistical Theory and Methodology in Science and Engineering, John Wiley & Sons, Inc., 1960 (See Chapter 10, Article 10.6, Table 10.5, pp. 262-264).

2-4 Mandel, J., and Paule, R. C., "Interlaboratory Evaluation of a Material with Unequal Numbers of Replicates", Analytical Chemistry, Vol. 42, No. 11, pp. 1194-1197, Sept. 1970; correction in Vol. 43, No. 10, Aug. 1971.

4-1 Draper, N. R. and Smith, H., Applied Regression Analysis, John Wiley & Sons, Inc., 1966.

APPENDIX A

MIL-HANDBOOK 5 GUIDELINE ON CREEP AND CREEP RUPTURE DATA

ANALYSIS

Approved as shown here at the
58th Coordination Meeting in
Portland, Maine

October, 1979

9.3.6 Creep and Creep-Rupture Data

9.3.6.1 Introduction - Creep is defined as the time-dependent deformation of a material under an applied load. It is usually regarded as an elevated temperature phenomenon, although some materials creep at room temperature. If permitted to continue indefinitely, creep terminates in rupture. (First stage or logarithmic creep exhibited by many materials at lower temperatures is not the subject of this section.) Creep in service usually occurs under varying conditions of temperature and complex (multiaxial) stress states, leading to an infinite number of stress-temperature-time combinations. Creep data for use in general design are usually obtained under conditions of constant uniform temperature and uniaxial-stress; this type of data is the subject of this section.

9.3.6.2 Terminology - The definitions presented below will be helpful in preparing creep-rupture data for inclusion in MIL-HDBK-5.

Creep - The time-dependent deformation of a solid resulting from force.

Note 1 - Creep tests are usually made at constant load and at constant temperature. For tests on metals, the initial loading strain, however defined, is not included.

Note 2 - This change in strain is sometimes referred to as creep strain.

Primary Creep - Creep occurring at a diminishing rate, sometimes called initial stage of creep.

Secondary Creep - Creep occurring at a constant rate, sometimes called second stage creep.

Tertiary Creep - Creep occurring at an accelerating rate, sometimes called third stage creep.

Creep Test - A creep test has the objective of measuring deformation and deformation rates at stresses usually well below those which would result in fracture during the time of testing.

Creep-Rupture Test - A creep-rupture test is one in which progressive specimen deformation and the time for rupture are both measured. In

general, deformation is much larger than that developed during a creep test.

Stress Rupture Test - A stress-rupture test is one in which time for rupture is measured, no deformation measurement being made during the test.

Total Strain - The total strain at any given time, including initial loading strain (which may include plastic strain in addition to elastic strain) and creep strain, but not including thermal expansion.

Loading Strain - Loading strain is the change in strain during the time interval from the start of loading to the instant of full-load application, sometimes called initial strain.

Plastic Strain During Loading - Plastic strain during loading is the portion of the strain during loading determined as the offset from the linear portion to the end of a stress-strain curve made during load application.

Creep-Strain - The time-dependent part of the strain resulting from stress, excluding initial loading strain and thermal expansion.

Total Plastic Strain - Total plastic strain at a specified time is equal to the sum of plastic strain during loading plus creep.

Creep Stress - The constant load divided by the original cross-sectional area of the specimen.

Elapsed Time - The time interval from application of the creep stress to a specified observation.

Creep Rupture Strength - The stress that will cause fracture in a creep test at a given time, in a specified constant environment.

Note: This is sometimes referred to as the stress-rupture strength.

Creep Strength - The stress that causes a given creep in a creep test at a given time in a specified constant environment.

Rate of Creep - The slope of the creep-time curve at a given time determined from a Cartesian plot.

Creep-Rupture Curve - The results of material tests under constant load and temperature; usually plotted as strain versus time to rupture. A typical plot of creep-rupture data is shown in Figure 9.3.6.2. The strain indicated in this curve includes both the

initial deformation due to loading and the plastic strain due to creep.

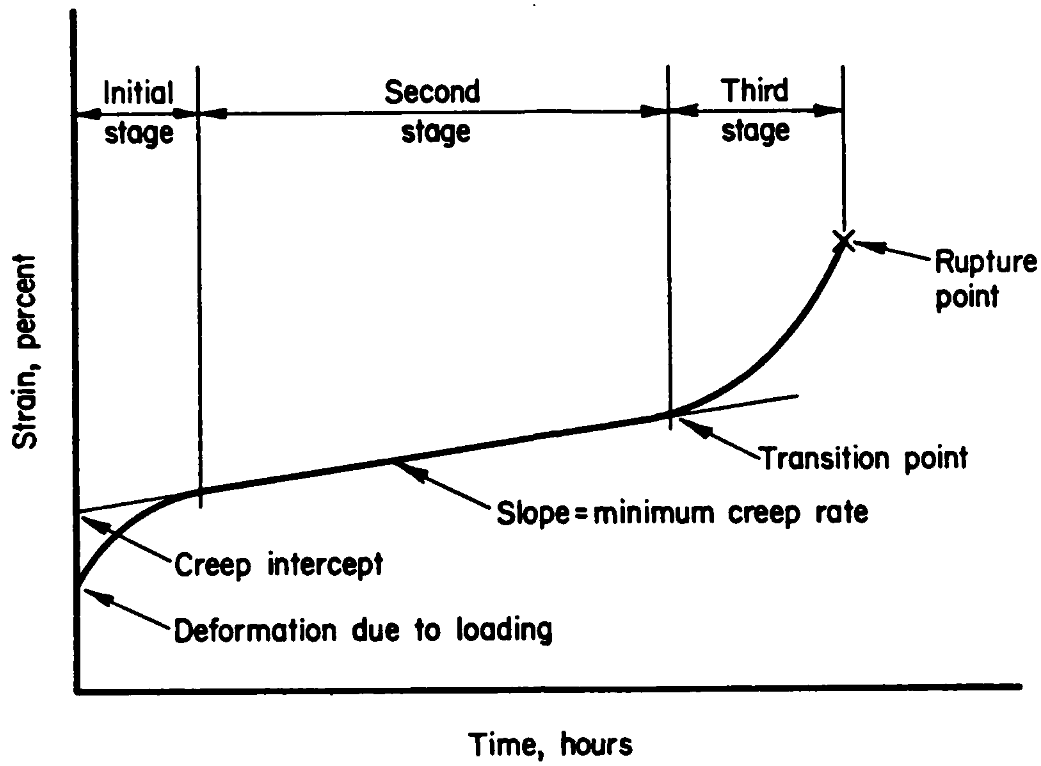

FIGURE 9.3.6.2. TYPICAL CREEP-RUPTURE CURVE

Master Creep Equation - An equation expressing combinations of stress, temperature, time and creep, or a set of equations expressing combinations of stress, temperature and time for given levels of creep.

Master Rupture Equation - An equation expressing combinations of stress, temperature, and time, that caused complete separation (fracture or rupture) of the specimen.

Isothermal Lines - Lines of uniform temperature on a creep or stress rupture curve.

Isostrain Lines - Lines representing constant levels of creep.

9.3.6.3 Data Generation - The following paragraphs provide guidelines on testing methods and designing an experimental matrix for developing creep and creep-rupture data.

Test Methods - Test methods must conform to ASTM E-139. However, it is recognized that this standard allows considerable latitude in procedures such that both the level and scatter of results can be significantly affected.

In case of significant difference in results from different testing sources, the following should be evaluated:

Material Condition (see Section 9.3.6.4)

Specimen Dimensions and Configuration (geometry effect)

Specimen Surface Preparation (residual stresses)

Specimen Alignment (concentricity, fixturing, load train, and loading method)

Temperature Control (number, type, and location of sensors, reference junction temperature control, monitoring and recording)

Extensometers (type, fixturing, and recording)

Strain Recording (records inelastic strain on loading and creates a record that can be evaluated for test stability)

Documentation (testing procedures)

General Laboratory Conditions, Personnel Qualifications, Calibration Intervals.

The submittor of a proposal should be prepared to provide documentation sufficient to permit a comparative evaluation of data. Inability to do so may cause the rejection of some of the associated data or the entire proposal.

Design of Experiments - A design of experiments approach to creep data development is highly recommended because it provides the maximum amount of useful data for the least expenditure of time and testing funds. If such an approach is not used, it is quite likely that several times as many test data will not serve as well in developing the desired mathematical models of creep behavior as data developed through design of experiments. This section is devoted to a description of the design of experiments approach which can be

used to develop regression models to mathematically portray creep rupture life and creep as a function of temperature and stress.

One method for planning testing is to develop a test layout in matrix form with temperatures in rows and expected creep lives in columns. Then through testing, simply fill out the blocks within the matrix. There should be a minimum of eight observations per isothermal line or twenty observations per Larson-Miller or other regression model. This ensures coverage of all of the conditions of interest. Further explanation of this method, by way of an example, is provided in Section 9.3.6.8.

Choosing the Number of Temperatures and Life Intervals - Before the test matrix can be formed, the interval sizes must be considered, first for temperature and then life.

(a) Temperature - A range of temperatures is usually required. For example, if the experiments must range from 1000 F through 1500 F, a choice must be made whether to perform tests at six levels (1000 F, 1100 F, 1200 F, 1300 F, 1400 F, 1500 F) or maybe at three levels (1000 F, 1300 F, 1500 F). The decision for this can be quite complicated and based on such phenomena as:

(1) The relative closeness of the isothermal lines

(2) Parallel or divergent isothermal lines

(3) The precipitation of secondary phases within the life ranges of interest.

However, this selection can be greatly simplified with very little user risk. Start with the lowest temperature and then choose the next temperature line such that at least one level of testing stress, on the log stress - log life plot, will be common to both temperatures. Then, proceed to the next temperature line, etc., ensuring like stress values on adjacent temperature levels.

(b) Life - Divide a log-life cycle into four equidistant segments. For example, between 100 hours and 1000 hours, the division would be approximately 180 hours, 320 hours, and 560 hours on the log-life scale. These divisions are far enough apart to insure a well-defined curve and a minimum overlap of data. To convert from temperature and life desired to temperature and

test stress requires that there be some prior knowledge of this relationship. If there is not prior knowledge, a series of "probe" tests must be made to locate the isothermal lines on a log-log plot.

Choosing the Number of Heats - Batch variations in chemistry, heat treating, etc., can cause considerable variations in the mechanical properties of an alloy. This difference is referred to as the heat-to-heat component as opposed to the within-heat components of variance.* Heat-to-heat standard deviation is usually 50-70 percent of the within-heat standard deviation. The root sum square of the two components of variance produce a measure of scatter about the regression that when added to the curve fitting error gives the regression parameter called the SEE (Standard Error of Estimate). The SEE is a product of the regression analysis; it is rarely determined as defined above. It is this parameter which fixes the design minimums about the regression estimates of the typical or mean values.

To make a mathematically sound decision on the minimum number of heats that should be used in a given analysis, it is necessary that an estimate of heat-to-heat and within-heat variance be known. This can usually be estimated from like alloys, or calculated from development data. Simulation has shown the following minimum number of heats to be satisfactory:

(1) When the heat-to-heat component of variance is less than 25 percent of the within-heat variance, use two heats equally for the sample sources.

(2) When the heat-to-heat component of variance is between 25-65 percent of the within-heat variance, use three heats equally.

(3) When the heat-to-heat component of variance is greater than 65 percent of the within-heat variance, use five heats equally.

Heats should be distributed randomly and essentially equally throughout the test matrix to ensure an unbiased heat distribution.

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*The within heat variance is the pooled variability of data from all heats, where the variability for each heat is calculated about its own average regression line. The heat-to-heat variance is calculated from the variability of each heat's average regression line about the overall average regression line of all heats. All heat average curves are assumed to be parallel in log life.

When regression models are developed from data that were not taken from an experimental model, the heats are rarely chosen randomly. Therefore, unless there are large samples of data in all areas of the regression matrix, this imbalance of heat sample sizes must be accounted for as described in Section 9.3.6.5. The order of testing must also be randomized so that anytime-oriented, operator-oriented, or machine-oriented effects are randomly distributed within the test matrix as described in Reference 9.3.6.3.

9.3.6.4 Data Collection and Interpretation - After a desired group of creep and/or creep-rupture data have been experimentally developed or isolated in pre-production files, it is necessary to carefully collect and interpret these data in accordance with the following guidelines:

Data Collection - For iso-strain creep, the collected data will include stress, temperature, modulus and plastic strain on initial loading, and strain-time pairs sufficient to define a curve. While the strain-time pairs will be only those for the iso-strains of interest after inelastic strain on loading has been included in the reported strain, it may be that the reported data may not correspond to the iso-strain levels. Consequently, the iso-strain-time pairs may be read from a smooth curve drawn through the values recorded during the test.

For rupture, the collected data will include stress, temperature, time-to-rupture, and percent elongation, and reduction of area. The percent elongation and reduction of area can then be used to define the rupture ductility curves or equations.

Data Interpretation - The state of the art for interpreting these types of creep and rupture data requires that a certain amount of judgement be allowed. The general approach will be to optimize one of several empirical equations that best follows the trend of the data, using life (or time) as the dependent variable. The independent variables will include stress and temperature for rupture and iso-strain creep curve and will also include strain for iso-strain creep curves.

Rupture ductility can be an exception to the above because of complex behavior and data scatter. At least a cautionary note should be given in the introductory material on the time and temperatures included in the rupture

data. Some materials exhibit such low elongation in certain time-temperature regions that normally reasonable values of design creep strain cannot be achieved without risk of fracture.

The interpretation of creep and rupture data should also include the variables that are reflected in the background data reporting requirements (discussed in the next subsection). Depending on the information content of the data and the type of variable, it may be desirable to develop a series of equations or to include additional physical variables in the regression analysis. The proposal should demonstrate that these additional variables have been evaluated and appropriately treated in the analysis.

The individual interpreting the data should also take note of the following special types of data, and consider the following recommendations on their use:

Specification Data - Virtually all alloys used for high-temperature applications are controlled and purchased by a process control variable generally called the "spec point." Therefore, there will often be large quantities of data available from quality control data records at the specification condition. The data will contain many heats and serve as an excellent measurement source in regression equations of what is referred to as "scatter." Therefore, in regression modeling, specification data are often the major source of the scatter measurements. The slope measurements must come from the experimental design matrix.

Specification data can also be used to (1) determine through analysis-of-variance techniques the fractions of the scatter due to heat-to-heat variations, etc., (2) determine through distribution analysis if the data are normal, log normal, etc., and (3) find out, if it is not normal, what transformation is required.

Outliers - These can be excluded only if the tests are demonstrably invalid or if the effect on the equation and the statistical parameters is unreasonable. Since the exclusion of outliers normally involves a certain degree of judgement, it should only be done by a knowledgeable and experienced individual.

Discontinued Tests - These can be included if longer lived, or excluded if shorter lived, than the average life of the data subset (lot, section thickness, etc.) to which they belong. (Also, see censored distributions in Section 9.3.6.5 on Data Analysis Procedures.)

Stepped-Tests - If the load on the specimen had been increased or decreased after the initial loading, this test result shall be excluded.

Truncating Data - Certain equations, notably the parametrics, are often unable to properly represent a mix of shorter and longer time data; these equations can severely over-predict creep and rupture lives less than ten to thirty hours. Similarly a preponderance of short time data can cause longer lives to be over-predicted. Eliminating such data requires truncating the data (or subset): this is done by removing all data above (or below) a fixed stress level, even though normally acceptable data are excluded.

Background Data Reporting - The significance and reliability of creep data generated at elevated temperatures for heat-resistant alloys are, to a major extent, a function of detailed factors which relate to the material, its processing, and its testing. Hence it is necessary to evaluate not only the property data, but also correlative information concerning these factors.

It is not possible to specify the individual items of correlation information, or the minimum thereof, which must be provided with elevated temperature property data to make those data properly meaningful. Individual alloy systems, individual product forms, and individual testing practices can all be quite unique with regard to associated information which should be provided with the data. A certain minimum amount of information is required for all data, including:

(1) Identity of alloy
(2) Chemical composition of the specific material tested
(3) Form of product (sheet, forging, etc.)
(4) Heat treatment condition
(5) Producer(s)

(6) Specification to which the product was produced

(7) Date when part was made.

Lack of such information is a sufficient basis for rejection of a particular data set.

In addition, it is vital that the individual submitting data consider those factors which contribute to the uniqueness of the alloy, the processing, and/or the testing, and give thought to information which is pertinent to that uniqueness. Thus, grain size can be a significant variable not only between cast turbine blades but within a single blade; thermomechanical working processes may result in significantly different properties (not only higher, but lower as well); and test specimen design can affect resultant data. It is mandatory that knowledgeable personnel be involved when data are submitted for evaluation and potential use. Any correlative data that can be provided will aid the analyst in identifying valid reasons for rejection of data which may not fit the trends of the other data (outliers). Such apparent outliers may be indicated through the analysis of between-heat variance as described in Section 9.3.6.5.

These examples illustrate the need for adequate information:

(1) Creep-rupture specimens are being machined from cast high-strength nickel-base alloy turbine blades. At center span location, specimens are 0.070 to 0.090 inch diameter, while at the trailing edge specimens are flat and 0.020 inch thick. The flat specimens are typically about one Larson-Miller parameter weaker than the round specimens, which is attributable both to the thickness effects of the thin specimens and to the finer grain size at the trailing edge. In addition, the trailing edge specimens exhibit more scatter. Hence, the availability of associated information is vital when considering data from specimens machined from cast turbine blades.

(2) Comparison of the creep-rupture properties of Waspaloy and Superwaspaloy shows that the latter is much weaker at temperatures approaching the upper bounds of utility of the alloy. The significantly lower properties at higher temperatures are attributed to a finer grain size of the Superwaspaloy and also

to a recovery process that may well be occurring at these temperatures. This alloy is subjected to extensive thermomechanical working, and some of the strengthening gained by the associated warm working is lost at the higher testing temperatures. This effect clearly indicates that processing history significantly affects the levels of mechanical properties and hence must be adequately documented when property data are submitted.

9.3.6.5 Data Analysis Procedures - After an acceptable data collection has been obtained and interpreted, it is possible to proceed in analyzing those data and developing mathematical models of creep and creep-rupture behavior. The objective of the procedures described in the following paragraphs is to calculate creep and rupture life as a function of test conditions and other significant variables. This calculation is done to provide an average curve and a measure of the expected variability about the average. The approach that is discussed involves regression analysis to optimize the fit of an equation to the data set. Linear regression analysis is described in Section 9.6.3. The following information provides guidelines in the application of regression analysis to creep and rupture data and recommends approaches to specific problems that are frequently encountered.

General - It is assumed that life or time is the dependent variable for the rupture or iso-strain creep equation analysis, respectively, and the logarithmic transformation of the dependent variable is normally distributed.

The data set will nearly always contain a variety of stresses and temperatures. If the data set is the product of a very well-balanced test design (see Section 9.3.6.3), good results may be obtained by independently fitting each temperature. Since this type of data set is often not available and the approach sacrifices the opportunity for interpolation, the discussion will assume that at least temperature and stress are used as independent variables.

In order to achieve good results, it may be necessary to consider other variables. Some variables are continuous physical variables that are incorporated into the regression variables, e.g., section size. Other

variables may occur as discrete subsets that require modifying the regression analysis (this is discussed under Subsets of Data). In such cases, it may be necessary to group the data per subset for data reporting if the regression analysis cannot easily accommodate the observed subsets.

Selection of Equations - For iso-strain and rupture time, as a function of stress and temperature, a number of relationships have been proposed. Some useful ones are:

(1) $\log t = c + b_1/T + b_2X/T + b_3X^2/T + b_4X^3/T$

(2) $\log t = c + b_1/T + b_2X + b_3X^2 + b_4X^3$

(3) $\log t = c + b_1T + b_2X + b_3X^2 + b_4X^3$

(4) $\log t = c + (T - T_a)(b_1 + b_2X + b_3X^2 + b_4X^3)$.

These are the Larson-Miller, Dorn, Manson-Succop, and Manson-Haferd, respectively, where

c is the regression constant

b_i are the coefficients (b_1 through b_4)

t is time

T is absolute temperature (T_a is the temperature of convergence of the iso-stress lives)

X is log S (stress).

While all of the forms may be used to model a data set with varying degrees of goodness of fit, experience and practice indicates the Larson-Miller relationship adequately models most materials and is usually the preferred equation form.

If none of these standard forms satisfactorily follow the data trends, various combinations of stress and temperature may be tried. For example, terms can be selected from a matrix obtained using the cross products of T^{-1}, T^{o}, T^{1} with S^{-1}, S^{o} and S^{1}. Methods for generalizing and applying these equations are discussed by M. K. Booker in a paper, "Regression Analysis of Creep-Rupture Data - A Practical Approach", published in Reference 9.3.6.5.

The exact form of the functions should reflect the data and reasonable boundary conditions. Quadratic, quartic, etc., can be expected to give poor boundary conditions, e.g., zero life at zero stress, and should be avoided. Extrapolation by users of the equation is inevitable (though it is

not recommended) so other general equations must be checked for unusual behavior beyond the data - this can be done in many cases by differentiating to obtain maxima and minima. In general, short times should give strengths approximately corresponding to tensile yield and ultimate strength; zero stress should predict infinite life.

Metallurgical instabilities and transition regions may present difficulties in some analyses. Methods for handling such problems have been discussed by L. H. Sjodahl in a paper, "A Comprehensive Method of Rupture Data Analysis With Simplified Models", published in Reference 9.3.6.5.

Optimum Fit - Guidelines for an optimum fit are:

(1) A minimum number of terms. With two independent variables (σ, T), six regression variables are reasonable, with each additional physical variable allowing two additional regression variables.

(2) Reasonable curve characteristics for material behavior, including extrapolation.

(3) Minimum standard error and maximum correlation coefficient (as long as 1 and 2 are not violated). Standard errors are typically between 0.1 and 0.2.

(4) Uniform deviations (see a later paragraph on Weights for a brief discussion of nonuniform deviations and their analytical treatment).

Subsets of Data - A nonnormal or multi-modal population, or an excessive standard error may indicate the presence of subsets. However, an apparently typical data set may contain subsets that should receive special consideration.

One type can be treated by adding physical variables to the regression analysis. For example, different thicknesses of sheet material may give different average lives. Including sheet thickness in the regression should not only improve the fit but also avoid the risk of misrepresenting the behavior of the material. Section thickness, distance from surface, and grain size, are other examples of subsets that can be treated as regression variables. Section thickness and distance from surface refer to the location of

the specimen in terms of the geometry of the original material, e.g., finish work thickness, final heat treat thickness, etc.

A second type is not typically subject to use as a regression variable. Examples of these are orientation (L, LT, and ST), or different heats (chemistry). A decision must be made whether to treat these as unique subsets to be analyzed separately (if properties are different) or as randomly distributed subsets. Orientation will usually be analyzed separately while heats will usually be randomly distributed subsets.

The theory of the treatment of randomly distributed subsets has been developed in Reference 9.3.6.3 while the application to lots of material (actually "heats" based on chemistry) is considered in a paper by L. H. Sjodahl in Reference 9.3.6.5. Treating subsets as random affects the calculation of both the average curve and the standard error. While the effect on the standard error may become insignificant as the number of subsets exceeds ten (depending on the relative contribution to the total standard error), the effect on the trend of the calculated average remains. Lots whose average lives are uniformly displaced (parallel) in logarithm of life, or are not significantly nonparallel, are discussed by L. H. Sjodahl in Reference 9.3.6.5. There is no known published reference for treating nonparallel lots. The data permitting, individual lots can be fitted, the within-lot variances pooled, and the average and variance of lot averages calculated for selected stress-temperature combinations. After calculating the total variance and the desired lower level tolerance limit* (X - ks) at each stress level curves can be drawn and, if desired, equations be fit to the X's and (X - ks)'s. It should be noted that the equation for (X - ks) is not likely to properly reflect uncertainty in coefficients that would be obtained by normal fitting procedures. Alternatively, all the data for nonparallel lots can be pooled and the variance weighted, providing sufficient lots are represented and the average curve is reasonably similar to that of the first approach.

Weights - Rupture and iso-strain creep curves will not normally require weights to obtain uniform variables. Analyses including strain as a

*Tolerance limits used here are one-sided and are normally developed for tolerance levels of 90 or 99 percent at a confidence level of 95 percent.

variable frequently will. Variables other than strain, temperature, and stress will require evaluation for uniform variance. A paper by L. H. Sjodahl, Reference 9.3.6.5, provides further discussion of weighting.

9.3.6.6 Preparation of Creep-Rupture Data Proposals - Creep-rupture proposals developed for review and possible inclusion in MIL-HDBK-5 should contain the following information and meet the associated criteria.

Data Reporting - The background information shall meet the requirements of Section 9.3.6.4. The test results shall be listed in a manner such that all data are identifiable in terms of the material and test background information, as well as the test conditions used in generating the data (see Section 9.3.6.7 for an example).

Analysis Reporting - The analysis report will display the following (see Section 9.3.6.5 for details):

(a) Trials - Equations tried and reason for rejecting.

(b) Data rejected - Reason.

(c) Best-fit details - Listing of data, calculated values, and deviations. All data are to be clearly traceable in terms of data reporting requirements.

(d) Standard error or total variance and correlation coefficient.

(e) Subset variance - If random subsets are used, report both the pooled within-subset variance and the between-subset variances as well as the total variances.

(f) Constants - Report the average regression constant and the regression constants for any subsets.

(g) Coefficients - Report the numerical value of the coefficient of each regression variable and its standard error.

(h) Equation - Exhibit the equation used; with the coefficients, b_i, traceable to the numerical listing in above item (g).

(i) Deviation - Exhibit plots of deviations in life vs. calculated life for each temperature and, in so far as possible, identify according to subsets. It is also possible to provide a summary table of deviations. As an example of iso-strain creep or rupture, divide the life range of the data in five equal

logarithmic increments and, for each temperature, give the algebraic sum of the deviation within that increment. If random subsets are used, the deviations summed are to be those from within the respective subsets.

(j) Data and Curve Comparison - Display the data against the calculated curves, giving both the average and, for proposal consideration, at least 99 percent lower level tolerance limits lines at 95 percent confidence level. Encode the data with symbols as in the deviation plots. Scale the coordinates such that the curves have an apparent slope of about -1.0. Use scales appropriate for the most significant form of the regression variable, usually log stress versus log life, with life (the dependent variable) on the abscissa and stress on the ordinate.

(k) Curve Extrapolation Tests - Exhibit the 99 percent probability of exceedance curve from one hour to 10^6 hours and the average curve for the corresponding stress levels. Representative curves may be used that include the extreme values of the independent variables as represented in the data.

The above recommendations apply to the incorporation of new creep and/or stress rupture curves in the Handbook. The incorporation of creep monographs in the Handbook has been discontinued. The creep monographs in the Handbook will be replaced as the data are reanalyzed and new analytically defined creep and stress rupture curves are developed.

9.3.6.7 Data Presentation - The presentation for MIL-HDBK-5 will include one or more pages of correlative information, equations, and curves as needed. The requirements on each will vary with the problem and should be reasonably obvious from the data, background information, and analytical results. The following data presentation is representative.

HS 188 RUPTURE

Alloy Designation:

Specification(s)	AMS 5608
Product Form	Sheet
Heat Treatment	2150 F anneal, rapid cool
Number of Vendors	One
Number of Lots	Ten

Specimen Description:

Type	Sheet
Gage Length	1.125 inches
Gage Width	0.250 inch
Gage Thickness	0.01 to 0.10 inch

Test Conditions:

Number of Test Laboratories	Three
Number of Tests	540

The lines shown on Figure 9.3.6.7 are limited to the available test times and temperatures, because extrapolation in time or temperature has definite risks. If extrapolation is necessary, the equation defining these lines should not be used below 1300 F or above 60 ksi. (These extrapolation limits will, of course, be different for each alloy.) The standard error of estimate (SEE) is given on Figure 9.3.6.7, along with the associated degrees of freedom. This parameter can be used to compute desired minimum curves, e.g., 95 percent, 99 percent, etc.

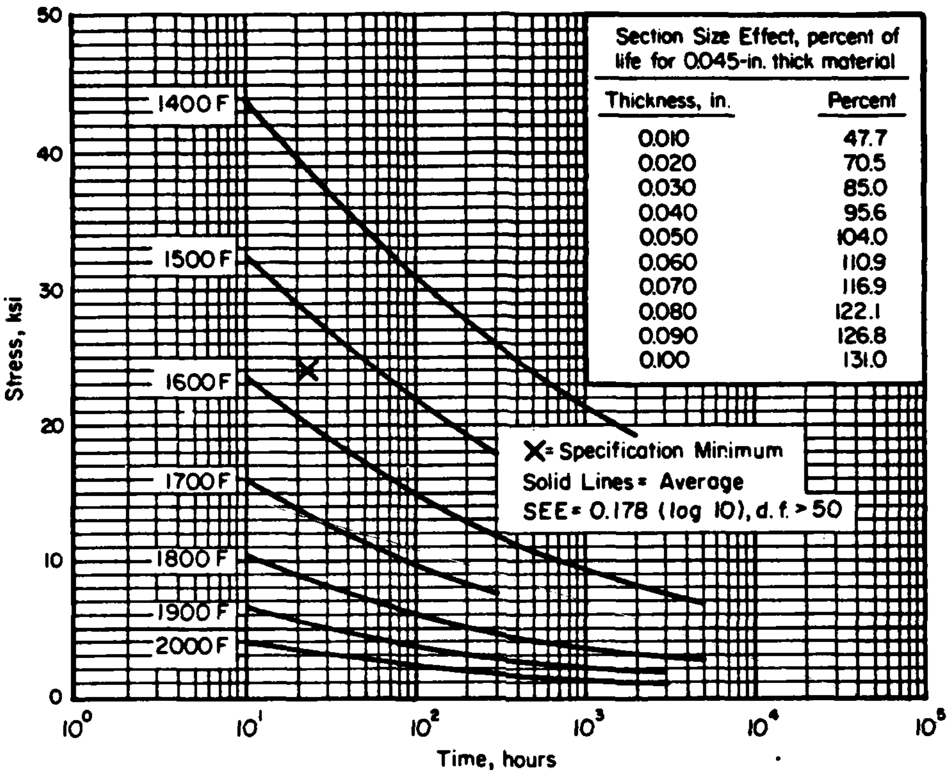

FIGURE 9.3.6.7. AVERAGE ISOTHERMAL RUPTURE CURVES FOR HS 188 SHEET

The coefficients, b_i, in the order given, and the average constant, c, define the equation of the average:

$$y = c + b_1/T + b_2X/T^2 + b_3X^3/T + b_4X^3T + b_5(2/m-0.006)^{0.4}$$

where

T is degrees Rankine

X is log (σ/9), where 9 is the inflection point in ksi

y is $\log_{10}t$

t is time to rupture in hours

m is thickness (X10^{-3} inch)

is stress in 1000 psi.

Average Constant (c) -17.525

Coefficients (b_i)

4.3257E+04 -2.1710E+07 7.3177E+02 -3.1103E-04 -1.3001E+00

Mean Values of Regression Variables

4.6704E-04   2.8074E-08   8.4749E-06   1.3323E+01   3.0104E-01

Within-Heat Variance	0.02713
Ratio of Between-To-Within-Heat Variance	0.16787
Intermediate Variance Ratio	1.01029

9.3.6.8 An Example of the Use of Experimental Design for the Purpose of Developing Regression Models - By a slight chemistry change and modification of the heat, the former Alloy 325 is now believed to have an increased stress rupture life of 20-30 percent. It is desired to fully characterize these properties over the 1600 F to 1900 F range. Average creep life is to be from 10 hours to 1000 hours.

Nineteen stress rupture tests from two heats of the new alloy averaged 37.4 hours at 30 ksi/1800 F, s = 0.150 (log base 10). Figure 9.3.6.8(a) is a log-log mean life plot of the predicted stress rupture properties of the modified Alloy 325 based on a predicted value. A 1750 F line has been added to the original plot. From this log-log plot, it can be seen that only three temperatures need to be tested because there are stress levels in common with the 1600 F line, and the same is true for the 1750 F and 1900 F lines.

Next, the three temperature lines are bracketed with the 10 hour-1000 hour life range. [See Figure 9.3.6.8(b)]. The stress levels are then chosen to give the desired life. There are 25 tests required doing it this way. All 25 could be run, or three tests could be randomly eliminated from the center cells of the matrix (see the circled cells). That would leave 22 tests, which is near the minimum of 20. These tests could then be run, added to the 19 specification data points @ 30 ksi/1800 F. This would now constitute the data set. Table 9.3.6.8 shows the results of a simulated sampling.

A Larson-Miller analysis of the data produced the curves seen in Figures 9.3.6.8(c) and (d). The data plotted with the temperature lines of Figure 9.3.6.8(d) confirm a good fit over the range of data. The approach described in this example can be used for any creep or rupture experimental design.

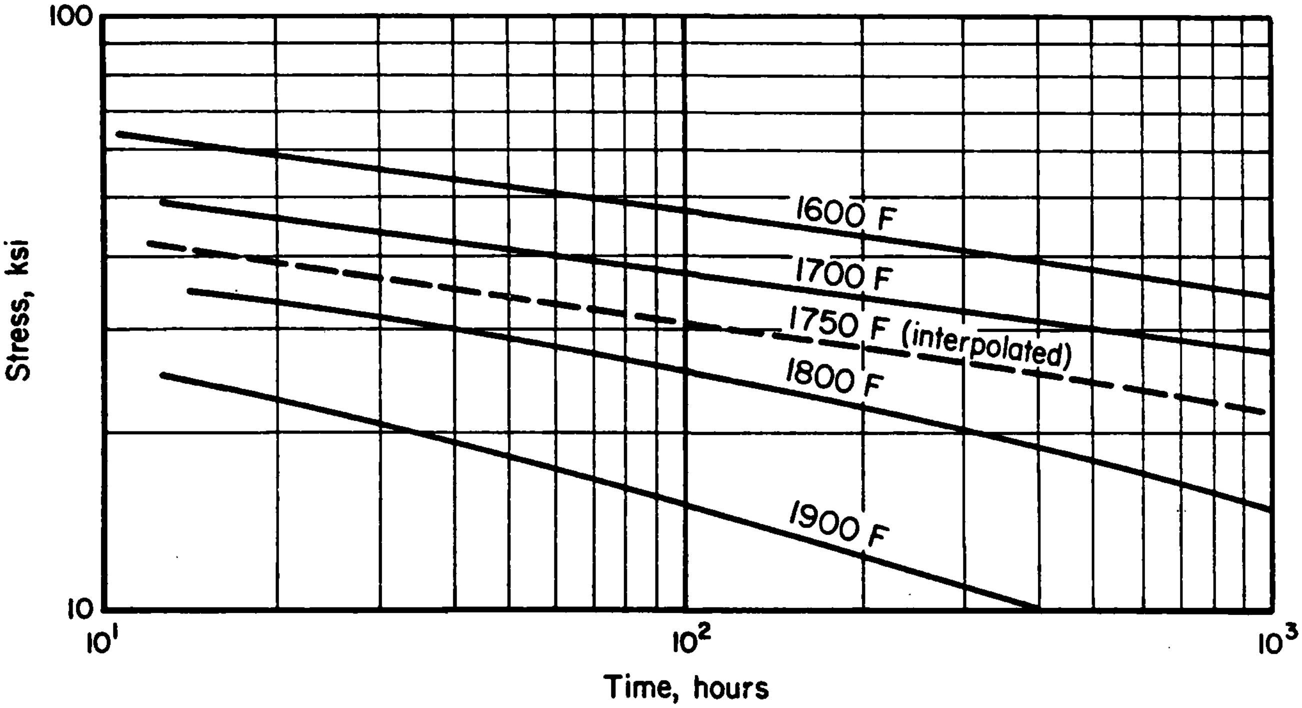

FIGURE 9.3.6.8(a). ESTIMATED STRESS RUPTURE CURVES FOR ALLOY 325 (MOD)

HOURS

TEMP		3	6	10	15	32	56	100	180	320	560	1000	3000	5600		F	
T1				63	59	54	52	48	45	42	39	36				1600	
T2				42	39	36	32	29	27	25	22	20				1750	
T3				25	22	20	17	15	12	10						1900	
T4																	
T5																	
T6																	
T7																	
T8																	

FIGURE 9.3.6.8(b). EXPERIMENTAL DESIGN MATRIX FOR CREEP RUPTURE

TABLE 9.3.6.8. RESULTS OF SIMULATED SAMPLING OF CREEP-RUPTURE DATA

ksi	1600 F	ksi	1750 F	ksi	1900 F
63	19.0 hrs.	42	8.8 hrs.	25	27.6 hrs.
59	11.1 hrs.	39	35.5 hrs.	22	23.9 hrs.
54	36.3 hrs.	36	52.3 hrs.	17	65.4 hrs.
52	170.7 hrs.	32	71.8 hrs.	15	140.3 hrs.
45	148.0 hrs.	29	121.9 hrs.	12	257.5 hrs.
42	376.0 hrs.	27	355.9 hrs.	10	623.5 hrs.
39	806.0 hrs.	22	389.0 hrs.	*	
36	878.0 hrs.	20	2912.4 hrs.	*	

*No interest.

SPECIFICATION DATA
@ 30 KSI/1800 F

Hours

41.4	33.1	70.5	36.1
16.5	27.4	37.5	34.9
35.0	33.4	48.6	74.2
33.6	51.3	29.0	47.5
32.6	42.7	26.4	

n = 19, X = 37.4, S(log 10) = 0.150

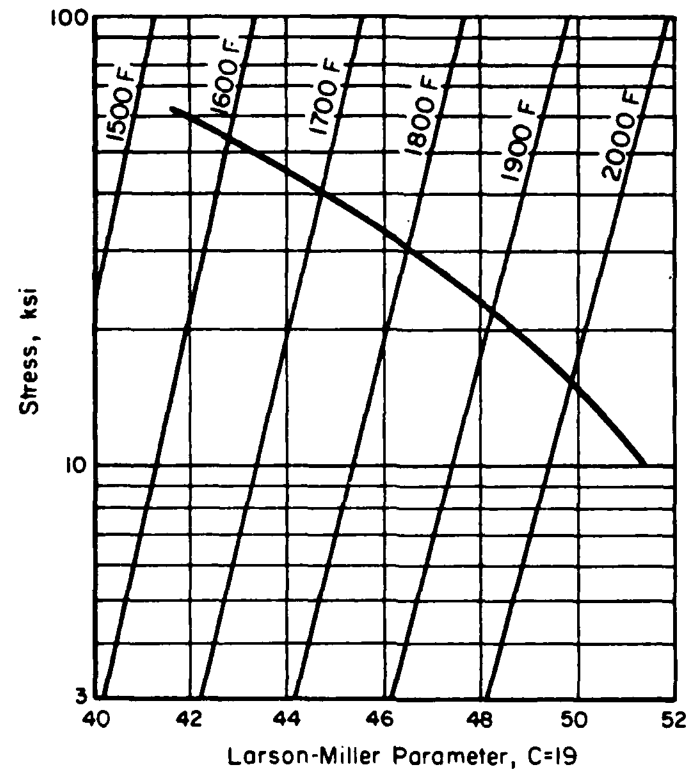

FIGURE 9.3.6.8(c). ALLOY 325 (MOD) STRESS RUPTURE TYPICAL LIFE

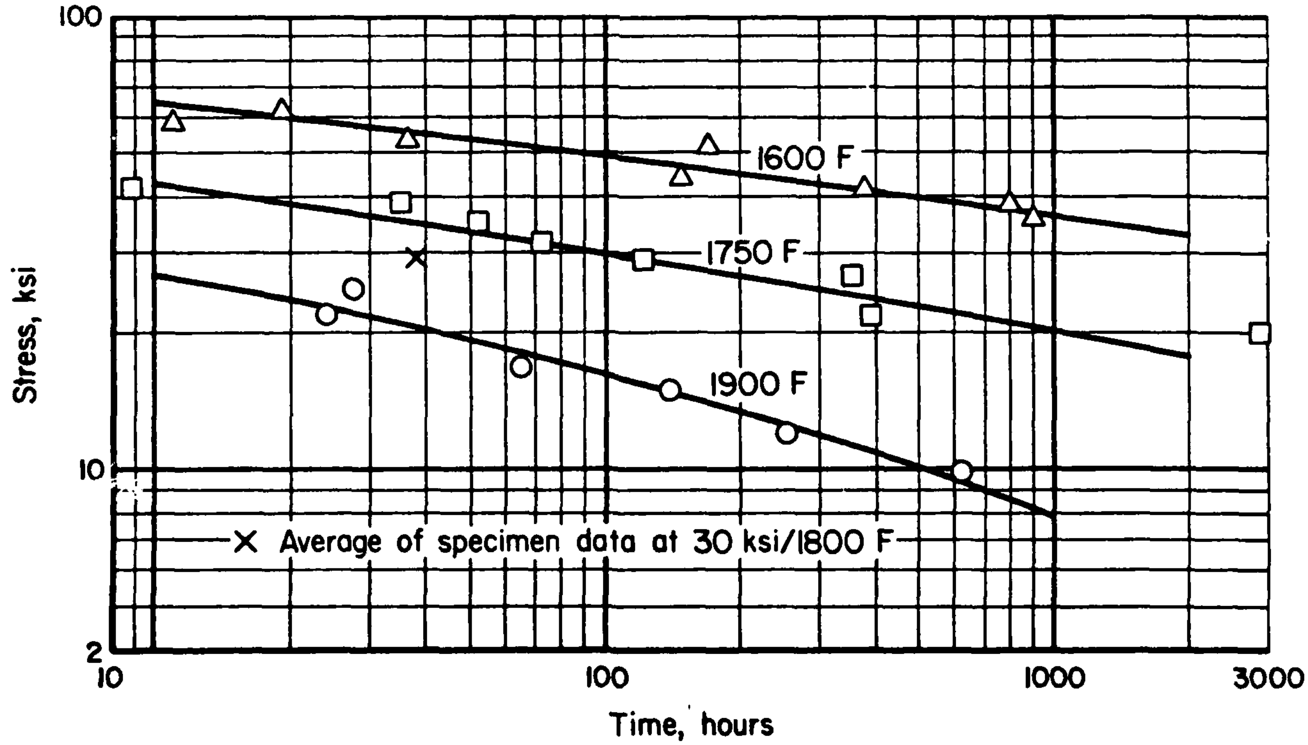

FIGURE 9.3.6.8(d). ALLOY 325 (MOD) STRESS RUPTURE TYPICAL LIFE

REFERENCES

9.2.2.3(a) Weissberg, A., and Beatty, G. H., "Tables of Tolerance-Limit Factors for Normal Distributors", Battelle Memorial Institute (December 1969).

9.2.2.3(b) Owen, D. B., "Tables of Factors for One-Sided Tolerance Limits for a Normal Distribution", Sandia Corporation (April 1958).

9.2.2.3(c) Owen, D. B., Handbook of Statistical Tables, Addison-Wesley (1962).

9.2.2.3(d) Guenther, W. C., Concepts of Statistical Inference, McGraw-Hill (1965).

9.3.2.3 Templin, R. L., et al, "Typical Tensile and Compressive Stress-strain Curves for Aluminum Alloys 14S-T, Alclad 14S-T, 24S-T, and Alclad 24S-T Products", Alcoa Research Laboratories Technical Paper No. 6 (1942).

9.3.2.4(a) Ramberg, Walter, and Osgood, William R., "Description of Stress Strain Curves by Three Parameters", NACA TN 902 (July 1943).

9.3.2.4(b) Hill, H. N., "Determination of Stress-Strain Relations from Offset Yield Strength Values", NACA TN 927 (February 1944).

9.3.3 Burt, C. W., et al, "Mechanical Properties of Aerospace Structural Alloys Under Biaxial-Stress Conditions", Battelle Memorial Institute, Technical Report AFML-TR-66-229 (August 1966).

9.3.4(a) "Manual on Fatigue Testing", American Society for Testing and Materials (1949).

9.3.4(b) "A Guide for Fatigue Testing and the Statistical Analysis of Fatigue Data", American Society for Testing and Materials ASTM STP 91-A (1963).

9.3.4(c) Aircraft Fatigue Handbook, ARTC Report W-76 (January 1957).

9.3.6.3 Mandel, J., and Paule, R., "Interlaboratory Evaluation of a Material with Unequal Numbers of Replicates", Analytical Chemistry, Vol. 42, No. 11, pp 1194-1197, Sept. 1970; correction in Vol. 43, No. 10, August 1971.

9.3.6.5 "Characterization of Materials for Service at Elevated Temperatures", Report No. MPC-7, Presented at 1978 ASME/CSME Montreal Pressure Vessel and Piping Conference, Montreal, Quebec, Canada, June 25-29, 1978.

9.4.1.3(a) "Military Standard, Fasteners, Test Methods, Test 4, Joint Shear Strength", MIL-STD-1312, Department of the Navy, Naval Air Systems Command, Custodians.

9.4.1.3(b) "Military Standard, Fasteners, Test Methods, Test 13, Double Shear Test", MIL-STD-1312, Department of the Navy, Naval Air Systems Command, Custodians.

9.4.1.3(c) "Military Standard, Fasteners, Test Methods, Test 20, Single Shear Test", MIL-STD-1312, Department of the Navy, Naval Air Systems Command, Custodians.

9.4.1.3(d) "Fastener and Joint Testing Procedure, Room Temperature", ARTC Report No. ARTC-33, Aerospace Industries Association of America, Inc.

9.4.3 Hood, D., et al, "An Investigation of the Generation and Utilization of Engineering Data on Weldments", The Boeing Company, Technical Report AFML-TR-68-268 (October 1968).

9.5.1.4.1 "Standard Method of Test for Plane-Strain Fracture Toughness of Metallic Materials", ASTM E399-72, ASTM Standards, Part 31 (1972).

9.5.1.5.1 Feddersen, C. E., "Evaluation and Prediction of the Residual Strength of Center-Cracked Tension Panels", Damage Tolerance in Aircraft Structures, ASTM STP 486, American Society for Testing and Materials (1971).

9.5.2.2(a) Sprowls, D. O., and Brown, R. H., "What Every Engineer Should Know About Stress Corrosion of Aluminum", Metals Progress, Vol. 81, No. 4 (April 1962).

9.5.2.2(b) Handbook on Corrosion Testing and Evaluation, Edited by W.H. Ailor, John Wiley and Sons, Inc., page 252 (1971).

9.6 Natrella, M. G., Experimental Statistics, National Bureau of Standards Handbook 91 (August 1, 1963).

9.6.1.1 Lilliefors, Hubert W., "On the Kolmogorov-Smirnov Test for Normality With Mean and Variance Unknown", American Statistical Association Journal, pp 399-402 (June, 1967).

9.6.4.1 Owen, D. B., "Factors for One-Sided Tolerance Limits and for Variables and Sampling Plans", Sandia Corporation Monograph SCR-607 (March, 1963).

APPENDIX B

A METHOD OF CALCULATING BETWEEN-HEAT VARIANCE AND AVERAGE EQUATION CONSTANT

Mandel and Paule's analysis equates two expressions for the within-heat variance, V_w. The first, from the regression, is $\sum\sum_{hj} (y_{hj}-y_h)^2/$ (N-H-k) where h and j are indices respectively of the heats $(1 \leq h \leq H)$ and of points in a heat $(1 \leq j \leq n_h)$, N is the total number of points (Σn_h), y_h is the predicted value of the dependent variable for heat h, and k is the number of independent variables.

The second expression for V_w is derived from the spacing between the parallel heat curves. This can be measured at any point along the curves; for convenience the constants, c_h, of the individual heat equations can be used. This is equivalent to adjusting all data to the test condition represented by zero values of all variables.

The variance of each c_h is given by $V_h = V_B + V_w/n_h$, where V_b is the between-heat variance, since the within-heat variability contributes to the observed scatter in the c_h. By introducing the term $\lambda = V_B/V_W$ (the ratio of between- to within-heat variance),

$$V_h = V_w\ (\lambda+1/n_h) = \sum_h (c_h-\bar{c})^2/(H-1) \text{ on the average,}$$

giving the second expression for V_w,

$$V_w = \Sigma[(c_h-\bar{c})^2/(H1)/(\lambda+1/n_h)] \quad .$$

These two expressions are made equal iteratively by adjusting the value of λ. Of course, being the ratio of variances, λ cannot be negative. λ is therefore set equal to zero when the calculated optimum value is negative (between-heat variance is negligible). V_B is now λV_W. Mandel and Paule further show that the weights used in calculating $\bar{c}$ (the average equation constant) are $w_h = 1/(\lambda+1/n_h)$, and the variance of $\bar{c}$ is equal to $V_w/\sum_h w_h$.